Optimal Scheduling for Profit Maximization Energy Storage Merchants Considering Market Impact Based on Dynamic Programming

Jian Liu, Rui Bo, Siyuan Wang, Haotian Chen

Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO 65409, USA

Abstract This paper analyzes how electricity merchants' market impact affects merchants' profit. Energy storage has long been studied for its role in maximizing profit, and merchant decisions are assumed to have no impact on market prices. However, the trading decisions of large-scale energy storage merchants (e.g., pumped storage hydro) will affect the market prices. This paper employs dynamic programming theory to investigate merchants' optimal economic dispatch considering the market impact and physical characteristics of storage systems. Our findings show that the State-of-Charge (SOC) based analytical solution significantly facilitates energy storage merchants' decision-making. The SOC range is segmented into three regions by two optimal SOC reference points, which depend on the available energy in storage, given prices, and market impact. By comparing the current storage SOC with the reference points, the merchant can get the corresponding optimal actions. We analytically show that if the merchant neglects the market impact power market, she will exaggerate her expectation profit when the price-taker and price-maker merchants have the same generating and pumping upper limits. The profit-maximizing merchant must, therefore, assay to balance the trade-off correctly between the intensity of market impact and the dispatched power. Our findings are verified by numerical simulation, and results demonstrate the ramifications for electricity merchants in energy arbitrage decisions.

Keywords: energy, electricity storage and dispatch, market impact, power market, dynamic programming

This article has been accepted for publication in Computers & Industrial Engineering. DOI: 10.1016/j.cie.2021.107212.

1 Introduction

The last decades have seen a boom in global renewable and clean energy resources (i.e., wind, solar, etc.) in the United States and worldwide. However, high uncertainty and intermittency of renewable energy and low forecast reliability are caused by the strong dependence on the weather (Qi et al., 2015, Zhou et al., 2019). Since electricity supply and demand must be matched in real-time, dealing with electricity surpluses or insufficiency is critical. As the imbalance of electricity can be dealt with storage, storing surpluses for future resale is a common strategy for commodities (Williams and 1991). With the control of electricity storage, the electricity merchant can not only regulate the intermittency of the energy and meet the demand of the market but also reduce costs and maximize arbitrage gains by controlling the stored energy, for example, buying electricity when the price is low and selling power when the price is high.

There are various energy storage technologies such as solar-thermal energy storage (Haslett, 1979), pumped storage hydropower (PSH) plant (Deane et al., 2010), battery storage (Cheng and Powell, 2018). Hydropower has long remained the largest source of renewable electricity generation, accounting for roughly 40% of U.S. renewable electricity generation in 2018, remaining the most significant contributor to U.S. energy storage, with an installed capacity of 21.6 GW or roughly 95% of all commercial storage capacity in the United States (Water Power Technologies Office, DOE, 2020). In our paper, we decide to discuss PSH and battery storage systems, both having their physical constraints, thus energy capacity of the storage facility, pumping/charging and generating/discharging capacity, facility pumping/charging and generating/discharging efficiency, and transmission efficiency, are all needed to take into consideration when modeling (Steffen and Weber, 2016； Zhou et al., 2016).

Traditional energy inventory research focuses on optimal policy or bidding decisions in modeling energy storage, some on PSH (Yeh, 1985, Löhndorf et al., 2013, Richmond et al., 2014), and some on battery storage (Lifshitz and Weiss, 2015, Zhou et al., 2016), allowing for the consideration of their physical constraints. According to the given market price, the merchant can decide on buying or selling electricity and obtaining the quantity according to the optimal policy. There are various methods to model the storage problem: online heuristic approach (Zhang and Wirth,2010), dynamic programming (Jiang and Powell, 2015), stochastic optimization in shaping energy (Powell and Meisel 2016), co-optimizing energy storage for energy arbitrage using convex relaxations (Hashmi et al., 2019), a control scheme for a wind farm with battery energy storage using sequential stochastic decision process (Kiedanski et al., 2019), charging and discharging algorithm for electric vehicles based on reinforcement learning (Lee et al., 2020), etc.

Most models (Jiang and Powell, 2011; Zhou et al., 2016; Zhou et al., 2019), however, assume that energy storage cannot be stored on a large-scale, and merchant's trading decisions do not affect market prices, which are called price-taker (PT). That is, according to the given price at each period, the merchant decides to buy or sell a certain amount of electricity at a price that is not affected by this merchant's activity. Nonetheless, the trading decisions of the large-scale energy storage (e.g., PSH) merchant will impact the market prices (Felix et al., 2012, Cruise et al., 2019), which terms she is a price-maker (PM). In detail, when the large-scale energy storage merchant chooses to buy electricity, the market load will increase, thus leading to the rising of market prices. On the contrary, selling power by a price-marker merchant will increase the supply, resulting in a decrease in market prices. We should note the difference between the optimal policies under two perspectives. Our paper addresses this critical issue.

In this paper, we focus on the application context of a price maker PSH owner or a battery owner, namely an electricity merchant (to facilitate the exposition, hereafter we use feminine pronouns when referring to the electricity merchant) using a storage strategy to manage electricity in the wholesale market. The storage facility's main features include the storage facility and pumping/generating capacity limits, a time-independent efficiency of energy inventoried in the storage facility dissipating during one period. We also include operational costs when trading electricity on the market.

With the motivations mentioned above, we aim to address the following two questions (1) What is the benefit of a price maker in electricity markets? Furthermore, (2) What is the difference in the optimal policy between both price taker and price maker? Toward that end, we first adjust the price by a linear function of the amount of the energy traded by the storage in the reward function to obtain the optimal policy considering the market impact. We investigate the electricity merchant's optimal decisions by employing the dynamic programming theory to maximize the profit according to the given available energy level/SOC in the storage, the current electricity prices, and the market impact. To the best of our knowledge, this is the first paper to solve the storage problem from the respective price-maker using dynamic programming.

Our study makes three principal contributions: First, for a price maker electricity merchant, the optimal trading policy at each decision time is deterministically determined by two optimal SOC reference points E_{t+1}^{p*} and E_{t+1}^{g*}, which depend on the available energy E_t in the storage, the current power prices P_t, and the *market impact*. Considering the efficiency loss or operating cost, the feasible energy storage level or SOC can be divided into three regions: for the positive electricity prices, if there is less energy in the storage than the respective reference point (i.e., $E_t < E_{t+1}^{p*}$), the merchant should buy power from the market and bring the SOC up to E_{t+1}^{p*}, and if there is more energy in the storage than the respective reference point (i.e., $E_t > E_{t+1}^{g*}$), the merchant should sell power to the market and bring the SOC down to E_{t+1}^{g*} as close as possible. However, if the stored energy is within the boundary set forth by the two reference points (i.e., $E_{t+1}^{p*} \le E_t \le E_{t+1}^{g*}$), the merchant should do nothing (i.e., stay in the idle mode). Under ideal condition, if both efficiency loss and operating costs are not considered, the feasible energy storage level or SOC can only be divided into two regions: buying-and-pumping and generating-and-selling.

Second, compared with the traditional study, when both price taker and price maker hold the same generating/pumping max capacity limits, the market impact will increase the cost of pumping, reducing the revenue generated in each period, which will reduce the optimal expectation profit. If the market impact is small, we will get similar optimal results, including unit commitment (UC) and economy dispatch (ED), as the scenario price-taker. When the market impact is large enough, the electricity merchant should reduce the power transaction quantity (i.e., amount of energy) at each period to lower the negative effect of market impact. The profit-maximizing merchant must, therefore, assay to perfectly balance the trade-off between the intensity of market impact and the power transaction quantity.

Third, we extend our research to consider how optimal scheduling is affected by water (energy) value at the end of the optimization horizon. Although both the value function of merchant and optimal SOC reference points in this extension are influenced by the value of water (energy) at the end of the optimization horizon, we can employ similar analytical procedures to derive insights for the profit-maximizing merchant.

We organize the remaining paper as follows: In Section 2, we review the literature on energy storage modeling and its impact on market prices. We then formulate the model in Section 3. Considering the market impact of the trading decision on prices, we apply them to the objective profit functions and give the optimal solution in Section 4. Section 5 verifies the proposed results based on synthesis data and real data of electricity prices from MISO (Midcontinent Independent System Operator, USA, 2020) which is one of Independent System Operators (ISOs) in North America and runs one of the largest electricity markets in the world. We conclude in Section 6 with a summary of our findings and some suggestions for future research.

2 Literature review

Our work is closely related to two aspects of the energy storage management and dispatch literature: energy storage modeling and market impact on the power market.

2.1 Energy Storage modeling

Yeh (1985) presents a general review of the mathematical models and simulations for reservoir operations. Brown *et al*. (2008) focus on using wind generators combined with PSH to meet the market electricity demand to minimize daily operating costs, while Castronuovo and Lopes (2004) apply the same storage system to figure out how to maximize the daily profit. All the above studies follow the principle that buying occurs when electricity demand is low, and electricity prices are low, selling occurs when electricity demand is high, and electricity prices are high (Deane *et al.*, 2010). Zhang and Wirth (2010) developed an online heuristic algorithm to smooth wind power variations with battery storage. Thus, the goal of the study is to find a policy that allows the electricity merchant to decide and compute the amount of energy (i.e., energy transaction quantity) to sell or buy based on the state of the environment period.

Aside from PSH systems, the battery is a more conventional but more expensive form of energy storage with some constraints. Considering the nature of the battery and other stochastic information like electricity prices, load demand, and regulation signals, Cheng and Powell (2018) propose a dynamic programming approach to solve the operation problem of the battery to charge and discharge to maximize arbitrage gains. Considering the battery life, Hashmi *et al.* (2018) propose an optimal arbitrage algorithm to control the number battery operations cycles to maximize the battery life and the arbitrage return. Nguyen *et al*. (2018) focus on the characteristic of charge or discharge efficiency of energy storage and propose nonlinear energy flow models based on nonlinear efficiency models and verify them by a Vanadium Redox Flow Battery (VRFB) system. To simplify our model, we take charge or discharge efficiency as a constant value.

Another stream of the literature concentrates on using storage to make better bidding decisions relative to periods in a market (e.g., Bathurst and Strbac 2003, Kim and Powell 2011, Löhndorf *et al.*, 2013, Jiang and Powell 2015). In this paper, we assume that any electricity offered to the market is accepted; thus, different from the above literature, we do not consider bidding problems. Wiser and Bolinger (2015), in some U.S. electricity markets, wind generators are treated as "must-run" in normal conditions, and 38% of the wind capacity developed in the U.S. in 2009 was sold through merchant agreements involving no bidding; thus, our assumption is acceptable.

2.2 Market impact on the power market

The present papers study the optimal control of storage by buying electricity when it is cheap and selling it when it is expensive to assume that the energy storage system is small relative to the market. The merchant's decisions do not affect market prices. (Secomand, 2010) propose the optimal inventory-trading policy with both space and injection or withdrawal capacity limits from the perspective of price-taker for a general case of the warehouse problem. Steffen and Weber (2016) consider the price-taker case of linear cost functions in the electricity market where electricity prices continue to change and determine how pumped storage devices maximize economic benefits. Following the perspective of price-taker, Zhou *et al*. (2016) model, energy arbitrage's price problem and take negative prices into account. Later, Zhou *et al.* (2019) extend the optimal structure when prices are negative and apply the stochastic dynamic programming to solve a co-optimization problem. From the perspective of price-taker, various methods are using to model the problem, such as dynamic programming (Zhou *et al.*, 2016; 2019), stochastic decision process (Kiedanski *et al.*, 2019), and mixed-integer linear programming (Chazarra et al., 2018; Zhan et al., 2020), these papers, however, did not consider market impact.

The large-scale energy storage (e.g., PSH) system may be captured in price arbitrage, then the trading decisions of the merchant are of sufficient magnitude as to have a market impact (Felix et al., 2012, Cruise *et* *al.*, 2019), which means that it serves as a price-maker. Felix et al. (2012) are the pioneers who offer an approach to storage valuation incorporating the market impact. Wei and Guan (2014) develop the optimal power storage control strategy for risk-neutral and risk-averse energy merchants to participate in both day-ahead and real-time market to maximize profit considering the impact of transaction decisions on the market. Steeger and Rebennack (2015) address price-maker hydropower companies' bidding strategy using game theory in the deregulated market. Cruise et al. (2019) apply the Lagrangian approach to optimize the deterministic model with stochastic prices to establish decisions and the forecast horizons when storage trading affects the market price. Different from the Lagrangian approach, we use dynamic programming to extend our storage model by allowing for the market impact when storage trading affects electricity prices (i.e., the respective price maker).

Unlike the studies described, our paper considers the effect of the market impact on the electricity trading decision. This approach is motivated by the observations reported in Felix et al. (2012) and Cruise *et al.* (2019). Considering the market impact will transform the traditional linear reward functions to nonlinear functions, it brings more challenges to achieve the analytical results.

3 Model Formulation

We consider an electricity merchant (PSH or battery owner) to use a storage strategy to manage electricity in an electricity wholesale market and thus buy and sell electricity. We work in discrete time, in which the merchant makes operational and trading decisions periodically over a finite horizon in each period $t \in \{1, 2, \cdots, T\}$. We assume the PSH storage (i.e., upper reservoir) has the maximum energy capacity $\bar{E}$ (e.g., the total energy which could be stored) and the minimum energy level $\underline{E}$, where, $\bar{E} > \underline{E} \geq 0$, which means the storage capacity is finite. We also assume the PSH storage or battery has generating or discharging and pumping or charging capacity constraints. We denote $\bar{Q}^p$ and $\underline{Q}^p$ as the pumping/charging upper limit and lower limit that can be purchased from to market in each period, $\bar{Q}^g$ and $\underline{Q}^g$ as the generating/discharging upper limit and lower limit that can be sold to the market in each period, respectively. This quantity is also referred to as the pumping or generating power max capacity if one does not consider the energy loss when pumping or generating the PSH. To maintain our model's tractability, we adopt the conventional assumption (Kim and Powell, 2011; Jiang and Powell, 2015) that $\underline{Q}^g = \underline{Q}^p = 0$.

Next, we will consider three types of efficiency with storage. The first of these is a fraction η_t, a time-independent efficiency, of energy inventoried in the storage facility, dissipates during one period, $1-\eta_t$ is the self-discharging rate of the battery, or the evaporation and leakage as well as spill rate of the PSH, equivalently, $\eta_t \in [0,1]$. The second of these are both α and β represent the efficiency of pumping mode and the efficiency of generating mode, where, $\alpha, \beta \in (0,1]$. Both $1-\alpha$ and $1-\beta$ represent the fraction of energy loss of pumping mode and generating mode, respectively. The third type of efficiency is ρ representing the fraction of transmission efficiency, that is, the ratio of electricity flowing out of the transmission line to that flowing into this line, so that $1-\rho$ is the line loss rate. Losses are incurred at the end of the transmission line in either direction, where, $\rho \in (0,1]$.

Based on the above discussion, we know that quantities $\bar{Q}^p / \alpha\rho$ and $\beta\rho \cdot \bar{Q}^g$ are the net pumping power capacity and gross generating power capacity. Different types of storage facilities can be modeled by varying the value of $\bar{Q}^p / \alpha\rho$ and $\beta\rho \cdot \bar{Q}^g$. If $\beta\rho \cdot \bar{Q}^g < \bar{E} - \underline{E}$ or $\bar{Q}^p / \alpha\rho < \bar{E} - \underline{E}$ represents slow storage, and the case $\beta\rho \cdot \bar{Q}^g \geq \bar{E} - \underline{E}$ or $\bar{Q}^p / \alpha\rho \geq \bar{E} - \underline{E}$ represents fast storage. In this paper, we target the optimal policy structure/decision rule for slow storage (i.e., a storage facility that cannot be fully emptied and filled up in one decision period) (Cruise *et al.*, 2019; Secomandi, 2010; Zhou et al., 2019). Fast storage is a special case for slow storage.

The electricity price in period t is denoted by P_t (dollars per unit energy). Both buying and selling prices at time t are shown by P_t conveniently for a price taker. The sequential levels of the price by a vector of $P = (P_1, P_2, \cdots, P_T)$. The decision for each period t is donated by q_t^g or q_t^p to represent the energy change (action) between period t and t+1 before accounting the efficiency loss. The quantity $q_t^g \cdot \beta\rho$ is the energy released from the storage to generate the power to sell to the market. The quantity $q_t^p / \alpha\rho$ is the energy/power bought from the market to pump the water to refill the upper reservoir of PSH or battery storage. During pumping/charging and generating/discharging, the electricity merchant needs to spend additional maintenance and operating costs. For the battery owner, the battery cycle life, a key issue when considering economic feasibility, varies between battery technologies and the operating conditions. In practice, the cost of maintenance and degradation is lower at first; after some point, costs increase much more rapidly. This paper lets c (dollars per unit energy) denotes the maintenance and operating cost for PSH or the battery's degradation cost. Under the current practice of MISO, the operating cost of PSH is close to zero (Huang et al.,2020). To maintain the tractability of our model, in this paper, we assume the energy storage has a linear operating cost of discharging/generating and pumping/charging.

Following the assumption of (Cruise et al., 2019), if the storage decision has a market impact, which means when the trading decisions of the electricity merchant are sufficiently influential (i.e., large-scale storage) to have a market impact on power prices, the price at which the merchant buys or sells energy can be approximated by a linear function of the amount of the electricity traded by the storage merchant. The adjusted prices are shown as follows:

$$\hat{P}_t = \begin{cases} (P_t + \lambda P_t \dfrac{q_t^p}{\alpha\rho}) \ (q_t^p \geq 0) \\ (P_t - \lambda P_t q_t^g \beta\rho) \ (q_t^g \geq 0) \end{cases} \tag{1}$$

In equation (1), the λP_t is a measure of the market impact of the storage on the price at time t. Parameter $\lambda \geq 0$ represents the *intensity of the market impact* of the merchant on power prices. If $\lambda = 0$, this special case corresponds to the situation of price-taker. $\hat{P}_t$ are the adjusted prices resulting from buying-and-pumping the storage by units of $q_t^p / \alpha\rho$ energy/power from the market and generating-and-selling the storage by units of $q_t^g \beta\rho$ energy/power to market, respectively.

Thus, the rewards function $R(q_t^p, q_t^g, \hat{P}_t)$ from performing decision pumping q_t^p and generating q_t^g, when the prices are $\hat{P}_t$ defined as follows from the respective of price-maker:

$$R(q_t^p, q_t^g, \hat{P}_t) = \begin{cases} -(P_t + \lambda P_t \dfrac{q_t^p}{\alpha\rho}) \cdot q_t^p / \alpha\rho - c(q_t^p / \alpha\rho) \ (q_t^p \geq 0) \\ (P_t - \lambda P_t q_t^g \beta\rho) \cdot q_t^g \cdot \beta\rho - c(q_t^g \cdot \beta\rho) \ (q_t^g \geq 0) \end{cases} \tag{2}$$

The first line in equation (2) represents the rewards when the electricity merchant releases the energy from storage to generate the power and sell to the market; for example, the $P_t \cdot q_t^g \cdot \beta \cdot \rho$ and $c \cdot q_t^g \cdot \beta \cdot \rho$ represent the revenue obtained and operating cost paid for her at time t for $q_t^g \cdot \beta \cdot \rho$ units of energy (power), respectively. The second line $P_t \cdot q_t^p / \alpha\rho$ indicates the cost when the electricity merchant buys power from the market to pump into the storage, and $c \cdot q_t^p / \alpha\rho$ shows the operating cost.

We denote E_t as the current energy/inventory in the storage or reservoir at the beginning t. The sequential levels of the storage by a $\hat{E}=(E_1, E_2, \cdots, E_T)$, where $E_i \in [\underline{E}, \bar{E}], \forall i \in \{1, 2, \cdots, T\}$. We define the feasible action/decision set based on the current energy level $E_t \in E$ as

$$\text{Action}(E_t) := \{(q_t^g, q_t^p) \in \mathbb{R} : 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g,\ q_t^g \le E_t - \underline{E},\ 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p,\ q_t^p \le \bar{E} - E_t\} \tag{3}$$

Here, the equation (3) expresses the maximum amount of power/energy that can be generated and pumped. The first two constraints show the upper boundary generating due to the upper limit and available energy in the storage. The third and fourth constraints define pumping's upper boundary because of the upper limit and the storage space capacity, respectively. Both binary variables U_t^g and U_t^p mean the unit commitment of generating and pumping in $[t,t+1)$. Without loss of generality, we have $U_t^p + U_t^g \le 1$ where, $U_t^g \in \{0,1\}$ and $U_t^p \in \{0,1\}$, which means the PSH cannot pumping and generating at the same period. If the PSH unit at the mode of offline, there is $U_t^p + U_t^g = 0$, that means the merchant does nothing (i.e., idle or offline). A ternary pumped storage system can simultaneously operate both the pump and generate (ANL/DIS-13/07). However, this is a different problem and beyond the scope of this study.

The merchant has three options: generating-and-selling, buying-and-pumping decisions, or do nothing, but at most, one of these decisions/actions is allowed. At the beginning of period t, the merchant knows the storage level E_t and the price P_t, then she decides that the quantity of power $q_t^g\beta\rho$ to sell to the market or $q_t^p / \alpha\rho$ to buy from the market will get the rewards $R(q_t^p, q_t^g, P_t)$. At the end of the period t, the storage self-loss happens, so the storage level at the start of the t+1 equals $\eta_t(E_t + q_t^p - q_t^g)$. Thus, we can get the following equation, which represents the storage energy balance or state transition from period t to t+1.

$$E_{t+1} = \eta_t(E_t + q_t^p - q_t^g) \tag{4}$$

The price-taker and the price-maker differ in whether an electricity merchant can impact the market. Hence, we analyze the price maker scenario and find the optimal decision rules in the next section.

4 Optimization and Analysis of the Price-Taker Storage

We first establish the objective profit functions in Section 4.1, Section 4.2 identifies the optimal solutions and insights from maximizing profit. Section 4.3 analyzes the effect of market impact for maximum expected profit.

4.1 Payoff Rewards and Objective Function

To maximize the profit, we assume for the merchant that all prices are known in advance so that the problem of controlling the storage is deterministic. The merchant makes operation and trading decisions periodically over a finite horizon in each period $t \in \{1, 2, \cdots, T\}$. Following the previous study (Zhou *et al*., (2016; 2019)), in this paper, we also model the merchant's storage strategy as a finite horizon Markov dynamic programming (DP). Each stage of the Markov DP corresponds to one period. The state variables in each stage t are E_t and P_t, the state at time t is denoted by $S(t) = S_t(E_t, P_t)$. The merchant's goal is to find the optimal decision rule π that maximizes the value function at stage 1 (i.e., initial stage) during the finite horizon $t \in \{1, 2, \cdots, T\}$.

As a price-maker (PM) merchant, the objective function, is shown as follows:

$$\max_{\pi} \sum_{t=1}^{T} E\left[\left((P_t - \lambda P_t q_t^g \beta\rho) \cdot q_t^g \cdot \beta\rho - c(q_t^g \cdot \beta\rho) - (P_t + \lambda P_t \frac{q_t^p}{\alpha\rho}) \cdot q_t^p / \alpha\rho - c(q_t^p / \alpha\rho)\right) | S(1)\right] \tag{5}$$

Subject to the capacity constraints $0 \le q_t^g \le \bar{Q}^g \cdot U_t^g;\ q_t^g \le E_t - \underline{E}; 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p;\ q_t^p \le \bar{E} - E_t$,

the unit commitment constraints $U_t^p + U_t^g \le 1$,

and the storage energy balance constraints $E_{t+1} = \eta_t (E_t + q_t^p - q_t^g)$,

where, $t \in \{1, 2, \cdots, T\}$.

In this paper, the function (5) ignores the discount factor, E is the expectation concerning E_1, P_1. Both E_1 and P_1 are the given initial level of the storage and the price in advance.

Let $V_t(S(t))$ denotes the value function in period t and state $S(t) = S_t(E_t, P_t) \in \hat{E} \times P$ .This function satisfies the Bellman equation. *Being a price-maker (PM) merchant*,

$$V(S(t)) = \max_{\text{Action}(E_t)} [R(q_t^p, q_t^g, \hat{P}_t) + E\left(V_{t+1}(S(t+1) | S(t)\right)] \tag{6}$$

Following the previous study (Secomandi, 2010; Zhou et al., (2016;2019)), we assume that any electricity left in the storage is worthless in the terminal period $T+1$. In this way, we will get

$V_{T+1}(S(T+1)) = V(E_{T+1}, \hat{P}_{T+1}) = 0$, which indicates the value of end energy (resp. water) in storage (resp. upper reservoir of PSH) equals zero. This assumption is interpreted as the merchant's only choice between generating-and-selling or doing nothing (offline or idle) in the last stage. E_{T+1} represents the energy level at the beginning of period T+1; it also equals the energy level at the end of period T.

The optimization problem of the merchant can be described to maximize $V(S(1))$. Equation (5) is a typical binary integer Markov DP function that is too complex to obtain the closed-form optimal analytical solutions. In this paper, we first replace the binary variables with an equivalent continuous decision variable and transfer the equation (5) to a traditional Markov DP to obtain the analytical optimal policy rule. Following Porteus (2002) and Zhou *et al.* (2016; 2019), we let $E_{t+1} = \eta_t(E_t + q_t^p - q_t^g) = \eta_t(E_t + \alpha_t)$ (i.e., $\alpha_t = q_t^p - q_t^g$) as the decision variable. We are using the action (decision) α_t for each period t to replace the previous decision q_t^g and q_t^p representing the energy storage level/SOC change between periods t to t+1 before accounting for the energy loss. Where $\alpha_t < 0$ represents the storage decrease due to the action of generating-and-selling, so the quantity of power sold to the market is $-\alpha_t \cdot \beta \cdot \rho$; $\alpha_t > 0$ indicates the storage increase due to the action of buying-and-pumping, so the quantity of power buy from the market is $\alpha_t / \alpha\rho$; $\alpha_t = 0$ shows the storage level does not change, or the merchant does nothing.

To obtain the optimal scheduling solutions, we also split the optimization in (6) into two sub-optimization problems corresponding to two different actions: buying-and-pumping from the market and the other generating-and-selling to the market in (1). Then, we find the optimal solution to each of these two sub-optimizations.

$$V_t(S(t)) = \begin{cases} V_t^p(S(t)) = \left\{ -P_t(1 + \frac{\lambda}{\alpha\rho}\alpha_t) \cdot \alpha_t - c \cdot \frac{\alpha_t}{\alpha\rho} + E[V_{t+1}(S(t+1) \mid S(t)] \right\} & (\alpha_t \geq 0) \\ V_t^g(S(t)) = \left\{ -P_t(1 + \lambda\beta\rho\alpha_t) \cdot \alpha_t\beta\rho + c \cdot \alpha_t \cdot \beta\rho \ + E[V_{t+1}(S(t+1) \mid S(t)] \right\} & (\alpha_t \leq 0) \end{cases} \tag{7}$$

Obviously, if $\alpha_t = 0$, there is $V_t^p(S(t)) = V_t^g(S(t))$. By using $\alpha_t = E_{t+1}/\eta_t - E_t$, and let E_{t+1} as the decision variable, then, the $V_t(S(t))$ in Equation (7) can be rewritten as follows:

$$\begin{cases} V_t^p(S(t)) = \left(E[V_{t+1}(S(t+1) \mid S(t)] - \frac{(P_t + c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) + \frac{(P_t + c)}{\alpha\rho}E_t - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) (\alpha_t \geq 0) \\ V_t^g(S(t)) = \left(E[V(S(t+1) \mid S(t)] - (P_t - c) \cdot (\frac{E_{t+1}}{\eta_t})\beta\rho + (P_t - c)E_t \cdot \beta\rho - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) (\alpha_t \leq 0) \end{cases} \tag{8}$$

The optimization problem can be segmented into both $\max_{E_{t+1}} V_t^p(S(t))$ and $\max_{E_{t+1}} V_t^g(S(t))$ subject to $\max\{-\bar{Q}^g, \underline{E}-E_t\} = -\bar{q}_t^g = \underline{\alpha}_t \le \alpha_t \le \bar{\alpha}_t = \bar{q}_t^p = \min\{\bar{Q}^p, \bar{E}-E_t\}$ in (8). Maximizing (8) can be approached by obtaining the optimal results to the equation (9) by removing the given state $S(t)$ (i.e., the given value E_t and P_t). Then, $V_t(S(t))$ should have the following results based on the Bellman equation (Puterman,1994).

$$\begin{cases} V_t^{P*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{(P_t+c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{P*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - \frac{(P_t+c)}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2_t\frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \\ V_t^{g*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t} - E_t)^2 \right) \\ \text{or } V_t^{g*}(S(t)) = \max_{\underline{E}\le E_{t+1}\le\bar{E}} \left(E[V_{t+1}^*(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t\beta^2\rho^2(\frac{E_{t+1}}{\eta_t})^2 + 2\lambda P_t\beta^2\rho^2\frac{E_{t+1}}{\eta_t}E_t \right) \end{cases} \quad \forall 1\le t\le T \quad (9)$$

We will analyze the optimal results based on equation (9) in the next section.

4.2 Optimization and Optimal Policy

To obtain the closed-form optimal policy/decision rule in equation (9), following the previous study (Kim and Powell, 2011, Porteus, 2002, Zhou et al., 2016), we know that for any $t \in \{1, 2, \cdots, T\}$, in every stage t, the value function $V_t(S(t))$ and $E[V_{t+1}(S(t+1)\,|\,S(t)]$ are concave in $E_t \in [\underline{E}, \bar{E}]$ for each given state $S(t) = S_t(E_t, P_t)$ and $P_t < \infty$ holds. Thus, we will get the following relationship:

$$\frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial {E_{t+1}}^2} = \frac{\partial\left(\frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial E_t}\cdot\frac{\partial E_t}{\partial E_{t+1}}\right)}{\partial E_t}\cdot\frac{\partial E_t}{\partial E_{t+1}} = \left(\frac{\partial E[V_{t+1}^*(S(t+1)\,|\,S(t)]}{\partial {E_t}^2}\cdot\left(\frac{\partial E_t}{\partial E_{t+1}}\right)^2\right) \le 0 \quad (10)$$

Therefore, we will get that the second-order derivatives of two sub-optimization problems in equation (9) are negative (i.e., $\frac{\partial V_t^{g*}(S(t))}{\partial {E_{t+1}}^2} < 0$, and $\frac{\partial V_t^{p*}(S(t))}{\partial {E_{t+1}}^2} < 0$), so, we can find the unique optimal solutions through the first-order condition. Thus, the optimal results of SOC are shown as:

Lemma 1: For the price-maker (PM) electricity merchant, let E_{t+1}^{g*} and E_{t+1}^{p*} are the optimal results in (9) for the price-maker scenario, respectively, as shown:

$$
\begin{cases}
E^{p*}_{t+1} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t)] - \frac{P_t + c}{\alpha\rho}(\frac{E_{t+1}}{\eta_t}) - \frac{\lambda P_t}{\alpha^2\rho^2}(\frac{E_{t+1}}{\eta_t})^2 + 2\frac{\lambda P_t}{\alpha^2\rho^2}\frac{E_{t+1}}{\eta_t}E_t \right) \\
\text{or } \left(\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - \frac{P_t + c}{\alpha\rho\eta_t} - 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t^2}E_{t+1} + 2\frac{\lambda P_t}{\alpha^2\rho^2\eta_t}E_t \right)\Bigg|_{E_{t+1}=E^{p*}_{t+1}} = 0 \\
E^{g*}_{t+1} = \arg\max_{\underline{E}\le E_{t+1}\le \bar{E}} \left(E[V^*_{t+1}(S(t+1)\,|\,S(t)] - (P_t - c)\cdot(\frac{E_{t+1}}{\eta_t})\beta\rho - \lambda P_t(\frac{E_{t+1}}{\eta_t})^2\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_{t+1}}{\eta_t}E_t\cdot\beta^2\rho^2 \right) \\
\text{or } \left(\frac{\partial E[V^*_{t+1}(S(t+1)\,|\,S(t)]}{\partial E_{t+1}} - (P_t - c)\cdot(\frac{\beta\rho}{\eta_t}) - 2\lambda P_t(\frac{E_{t+1}}{\eta_t})\frac{1}{\eta_t}\cdot\beta^2\rho^2 + 2\lambda P_t\frac{E_t}{\eta_t}\cdot\beta^2\rho^2 \right)\Bigg|_{E_{t+1}=E^{g*}_{t+1}} = 0
\end{cases}
\tag{11}
$$

In equation (11), we can safely draw that there exists two optimal reference points/functions E^{p*}_{t+1} and E^{g*}_{t+1} depend on the current energy storage E_t , the given price P_t, and the intensity of market impact λ. In equation (11), when market impacts are *not* considered (i.e., $\lambda=0$), which becomes a special case for the scenario of price-taker, we obtain the following relations:

$$
E^{p*}_{t+1(\lambda=0)} = E^{p*}_{t+1(PT)} \quad \text{and} \quad E^{g*}_{t+1(\lambda=0)} = E^{g*}_{t+1(PT)} \tag{12}
$$

Assume we ignore the market impact of the merchant's trading decision on price. In that case, the price maker scenario becomes a particular case of the price-taker, then we will get the same optimal decisions. It means that the profit-maximizing electricity merchant, the optimal trading policy at each decision time depends on the forecasted price, the available energy in the storage, and the market impact. Based on the above discussion, the corresponding optimal solutions are given in the following proposition (All proofs are given in Appendix A)

Proposition 1: For every stage $t \in \{1,2,\cdots,T\}$, and positive forecasted electricity price $\hat{P}_t \in P$ (resp. negative forecasted electricity price), when $0 \le \lambda \le \bar{\lambda}^{(p,g)}_t$[1], there exist a unique optimal storage level relationship $\underline{E} \le E^{p*}_{t+1} \le E^{g*}_{t+1} \le \bar{E}$ (resp. $\underline{E} \ge E^{p*}_{t+1} \ge E^{g*}_{t+1} \ge \bar{E}$), which depend on the state of $S(t)$ (i.e., current storage E_t , the price P_t), where $\bar{\lambda}^{(p,g)}_t = \left(\frac{P_t + c}{\alpha\rho} - (P_t - c)\beta\rho\right) \Big/ 2P_t(\frac{1}{\alpha^2\rho^2} - \beta^2\rho^2)\left(E_t - \frac{\underline{E}}{\eta_t}\right)$.

In this paper, we only target the positive electricity prices. Thus, an optimal decision in each state

---

[1]Large-scale storage/PSH capacities are around 1GW to 2GW. Considering a large competitive market (such as MISO which has roughly 190 GW installed capacity) and the limited presence of locational market power due to strong transmission and market monitoring, we focus on analyzing a relatively small market impact.

$S(t) = S_t(E_t, P_t) \in \hat{E} \times P$, can be specified as follows:

Case 1: If $\alpha\beta\rho^2 < 1$ (with efficiency loss) or $c \neq 0$ (with operating cost), then there is $E_{t+1}^{p*} \leq E_{t+1}^{g*}$, the feasible storage level or SOC can be divide into three regions: buying-and-pumping, generating-and-selling, and do nothing (or idle/offline).

$$\alpha_t^*(E_t,\hat{P}_t)=\begin{cases}\min\{E_{t+1}^{p*} - E_t, \bar{Q}^p\}(\text{buy and pump energy up to } E_{t+1}^{p*}) & E_t \in [\underline{E}, E_{t+1}^{p*}) \\ 0 \quad (\text{keep energy unchanged}) & E_t \in [E_{t+1}^{p*}, E_{t+1}^{g*}] \\ \max\{E_{t+1}^{g*} - E_t, -\bar{Q}^g\}(\text{generate and sell energy down to } E_{t+1}^{g*}) & E_t \in (E_{t+1}^{g*}, \bar{E}]\end{cases} \tag{13}$$

Case 2: If $\alpha\beta\rho^2 = 1$ (i.e., without efficiency loss) and $c = 0$ (without considering operating cost), then there is $E_{t+1}^{p*} = E_{t+1}^{g*} = E_{t+1}^*$ and the feasible storage level or SOC can be divide into only two regions: buying-and-pumping and generating-and-selling.

$$\alpha_t^*(E_t,P_t)=\begin{cases}\min\{E_{t+1}^* - E_t, \bar{Q}^p\}(\text{buy and pump energy up to } E_{t+1}^*) & E_t \in [\underline{E}, E_{t+1}^*] \\ \max\{E_{t+1}^* - E_t, -\bar{Q}^g\}(\text{generate and sell energy down to } E_{t+1}^*) & E_t \in [E_{t+1}^*, \bar{E}]\end{cases} \tag{14}$$

The first part of proposition 1 indicates that the price maker merchant also has three decision choices: generating, pumping, and idle (or offline). By comparing the current storage SOC with the optimal reference points, the merchant can schedule the corresponding optimal actions. If E_t less than E_{t+1}^{p*}, then the electricity merchant (i.e., PSH owner) should buy the power from market, then pump water and bring the SOC level up to E_{t+1}^{p*} as close as possible, and if E_t is larger than E_{t+1}^{g*}, the electricity merchant should release the water from the upper reservoir, generate and sell power to market then result in the SOC level down to E_{t+1}^{g*} as close as possible, however, if the stored energy is within the boundary set forth by the two reference points (i.e., $E_{t+1}^{p*} \leq E_t \leq E_{t+1}^{g*}$), then keep the SOC unchanged or the electricity merchant should do nothing, respectively.

The second part of proposition 1 indicates that without considering both operating cost and efficiency loss, there exists one optimal SOC threshold function E_{t+1}^* depends on the current available energy in storage E_t, the prices P_t, and the market impact λ. The merchant should generate-and-sell the power to market and down the SOC level to E_{t+1}^* or buy-and-pump the energy to the storage and result in the SOC up to E_{t+1}^*, respectively. Based on the two parts of proposition 1, we will get the following insight.

Managerial Insight 1. *For a price maker electricity merchant, the optimal trading policy at each decision time depends on the given power price, the available energy in the storage, and the intensity of market impact. From the respective price-maker to maximize the profit, the SOC range is segmented into three regions by two optimal SOC reference points, which corresponds to one of three distinct actions.*

These results for the electricity merchants bear a critical implication. Because a profit-maximizing electricity merchant only needs to compare the real-time storage SOC with the reference points only, she can get the corresponding optimal actions. This insight helps explain the observation and intuition that the merchant should pump (resp. generate) at full capacity when the prices are low (resp. high) if the SOC constraints do not bind, respectively.

4.3 Market Impact Analysis

Compared with the traditional study (i.e., without considering the market impact), although we still get similar insights that the feasible SOC can be divided into three regions or two regions, the merchant market impact affects the objective function. When other parameters remain the same except the market impact, based on the above discussion and assumption, the corresponding optimal solution is given in the following proposition (See Appendix A (Proof of proposition 2)).

Proposition 2: For every stage $t \in \{1, 2, \cdots, T\}$, and the forecasted positive price P_t, when both price-taker merchant and price-maker merchant have the same pumping and generating upper limits, the optimal value function and maximum profit have the following relations:

$$\begin{cases} E[V^*_{t+1(\lambda=0)}(S(t+1) \mid S(t)] \geq E[V^*_{t+1(\lambda>0)}(S(t+1) \mid S(t)] \\ \max\limits_{\pi} \sum\limits_{t=1}^{T} E\left[R(q_t, \hat{P}_t)_{(\lambda=0)} \mid S(1) \right] \geq \max\limits_{\pi} \sum\limits_{t=1}^{T} E\left[R(q_t, \hat{P}_t)_{(\lambda\geq 0)} \mid S(1) \right] \end{cases} \tag{15}$$

Proposition 2 shows that if the electricity merchants' trading decisions are of sufficient magnitude to have a market impact, whose trading decisions will affect the power prices --- when merchants choose to buy electricity, the market load will increase, leading to rising market prices; on the contrary, selling power by a price-marker merchant will increase the supply, resulting in a decrease in market prices--- which influence decision in return. Therefore, with the increases in market impact, the electricity merchant will obtain less profit when price-taker merchants and price-maker merchants have the same generating and pumping limits. Based on the results in proposition 2, we will gain the following insight.

Managerial Insight 2. *If the electricity merchant ignores the market impact power market, she will exaggerate her expectation profit theoretically when the price-taker merchants and price-maker merchants have the same generating and pumping limits offered to ISOs.*

These findings are consistent with the reported results by Felix *et al.,* (2012) and Cruise (2019), which means the merchant will decrease the expected profit with the increasing market impact. We propose our managerial insights and optimal scheduling to employ dynamic programming in this section based on the forecasted price. Next, we will verify the proposed analytical results through a case study.

5 Numerical Simulation

In this section, we first validate the proposed methods and results via two cases to express the analytic findings' characteristics and compared them with the mixed-integer linear programming based on synthesis data in section 5.1. Further, Section 5.2 uses real data from MISO (Midcontinent Independent System Operator, USA, 2020) electricity prices to indicate optimal insights.

5.1 Case Study and Comparison

For simplicity, we use two small cases to show the process detail of the proposed methods. The following cases provide the conditions under which we can get the corresponding optimal analytical results.

Case 1: In this case, we assume there are three time periods ($T=3$). At each period, the power price takes one of the values in set $P_t=\{p^M,p^L,p^H\}=\{5,2,10\}$. For simplify, we assume the storage energy capacity cannot refill it fully in one time period, but fewer than two time periods, it holds that $\underline{E}+\bar{Q}^p\leq\bar{E}$ and $\underline{E}+2\bar{Q}^p\geq\bar{E}$. We also assume the storage can sell it empty in one period (i.e., $\bar{E}-\underline{E}\leq\bar{Q}^g$). We assume the storage capacity is 10 (i.e., $\underline{E}=0,\bar{E}=10$), the generating/discharging max capacity is 12 and the pumping/charging max capacity is 7. For simplicity, let the operating cost be zero, and the pumping/charging, generating/discharging, self-discharging, and transmission efficiencies are one. To verify the effect of market impact, in this case, we assume the merchant's market impact parameter $\lambda=0.05$. On this basis; we use backward dynamic programming to get the following optimal policy and results:

In Stage 3: The value function is shown as follows:

$$V_3=-[p^H+\lambda p^H(\underline{E}-E_3)\beta\rho](\underline{E}-E_3)\beta\rho=-[10+0.5(\underline{E}-E_3)](\underline{E}-E_3)=10E_3-0.5E_3^{\,2}, E_3\in[0,10]$$

In Stage 2: By using E.q. (11) for price maker merchant, we will get the following results:

$$E_3^{g*}=E_3^{p*}=E_3^{*}=\underset{E_3\in[\underline{E},\bar{E}]}{\arg\max}\{V_3^{*}-p^L E_3-\lambda P^L[E_3-E_2]^2\}=\left(40+E_2\right)/6 \tag{16}$$

Following proposition 1, we will get the following optimal action at stage 2.

$$\alpha_2^{*}=\begin{cases}\min\{\frac{40+E_2}{6}-E_2,\bar{Q}^p\}=\frac{40-5E_2}{6} \text{ (buying and pumping up to } E_3^{*}) \text{ if } E_2\in[0,\frac{40+E_2}{6})\\ \max\{\frac{40+E_2}{6}-E_2,-\bar{Q}^g\}=\frac{40-5E_2}{6} \text{ (generating and selling down to } E_3^{*}) \text{ if } E_2\in[\frac{40+E_2}{6},10]\end{cases} \tag{17}$$

Thus, the optimal value functions in stage 2 are shown as follows:

$$V_2^{*}=\max(R_2+V_3^{*})=\left(320+40E_2-E_2^{\,2}\right)/12 \text{ if } E_2\in[0,10]$$

In Stage 1: By using E. q. (11) for price maker merchant, we will get the following results:

$$E_2^{p*}=E_2^{g*}=E_2^{*}=\underset{E_2\in[\underline{E},\bar{E}]}{\arg\max}\{V_2^{*}-p^M E_2-\lambda P^M[E_2-E_1]^2\}=\max\{0,\left(3E_1-10\right)/4\} \tag{18}$$

Following proposition 1, we will get the following optimal action at stage 1.

$$\alpha_1^{*}=\begin{cases}-E_1 \text{ (generating and seling up to 0)} & \text{if } E_1\in[0,10/3)\\ \frac{3E_1-10}{4}-E_1=\frac{-10-E_1}{4} \text{ (generating and seling down to } \frac{3E_1-10}{4}) & \text{if } E_1\in[10/3,10]\end{cases} \tag{19}$$

Thus, the optimal value functions at stage 1 are shown as follows:

$$V_1^{*}=\max(R_1+V_2^{*})=\begin{cases}(5E_1-0.25E_1^{\,2})+80/3 & \text{if } E_1\in[0,10/3)\\ \frac{700+60E_1-E_1^{\,2}}{64}+\left(320+40\cdot\frac{3E_1-10}{4}-\left(\frac{3E_1-10}{4}\right)^2\right)\Big/12 & \text{if } E_1\in[10/3,10]\end{cases}$$

We will get the following optimal results:

1) If $E_1=1$ (The initial SOC in the storage)

Stage 1: If $E_1=1$,(action 1: generate and sell, $\alpha_1^{*}=-1$), then $E_2=0$;

Stage 2: If $E_2=0$,(action 2: buy and pump, $\alpha_2^{*}=40/6$), then $E_3=40+E_2/6=40/6$;

Stage 3: If $E_3 = 40/6$,(action 3: generate and sell, $\alpha_3^* = -40/6$), then $E_4 = 0 = \underline{E}$.

The optimal value at stage 1 is shown as

$$V_1^* = (5E_1 - 0.25E_1^{\ 2}) + 80/3 = 31.4167 .$$

2) If $E_1 = 5$ (The initial SOC in the storage)

Stage 1: If $E_1 = 5$,(action 1: generate and sell, $\alpha_1^* = -3.75$), then $E_2 = 3E_1 - 10/4 = 1.25$;

Stage 2: If $E_2 = 1.25$,(action 2: buy and pump, $\alpha_2^* = 33.75/6$), then $E_3 = 40 + E_2/6 = 41.25/6$;

Stage 3: If $E_3 = 41.25/6$,(action 3: generate and sell, $\alpha_3^* = -41.25/6$), then $E_4 = 0 = \underline{E}$.

The optimal value at stage 1 is shown as

$$V_1^* = \left(700 + 60E_1 - E_1^{\ 2}\right)/64 + \left(320 + 40\cdot(3E_1 - 10)/4 - \left((3E_1 - 10)/4\right)^2\right)/12 = 45.9375 \cdot$$

Cases 2: In this case, we will show the difference in the optimal policy and corresponding results between both price-taker and price-maker. Following Case 1, let the operating cost be one(i.e., $c = 1$), the pumping and generating efficiencies be 0.9 (i.e., $\alpha = \beta = 0.9$), self-discharging and transmission efficiencies be one (i.e., $\rho = \eta = 1$), while other parameters (i.e., pumping and generating upper limits) remain the same. We assume $\lambda = 0$, $\lambda = 0.01$, and $\lambda = 0.02$ corresponding to different market impact in trading (See Appendix B). Then, the optimal results are shown in Table 1.

Table 1: Optimal Results with Market Impact

	$\lambda=0, E_1=1$	$\lambda=0, E_1=5$	$\lambda=0.01, E_1=1$	$\lambda=0.01, E_1=5$	$\lambda=0.02, E_1=1$	$\lambda=0.02, E_1=5$
(E_3^{g*}, E_3^{p*})	(10,10)		(10,10)		(10,10)	
(E_2^{g*}, E_2^{p*})	$(3,3)$		$\left(\frac{0.2272+0.081E_1}{0.13048}, \frac{10E_1/81+459/1500}{0.2854}\right)$		$\left(\frac{0.72+0.162E_1}{0.262}, 0\right)$	
α_3^*	-10	-10	-8.5	-10	-8	-10
α_2^*	7	7	7	5.16	7	5
α_1^*	2	-2	0.5	-0.16	0	0
V_1^*	44.34	64.87	34.1	55.7	28.68	46.9

To verify the proposed method's effectiveness, we also get the optimal results for the above two cases using the traditional mixed-integer linear programming (MILP) model (Chazarra et al., 2018; Zhan et al.,

2020) and compare the optimal results that obtained through Markov DP (proposed method this paper) and MILP (traditional method). Following the discussion and assumption in Section 3, the traditional MILP model for a price-maker (PM) merchant is shown as:

$$\sum_{t=1}^{T}\left((P_t - \lambda P_t q_t^g \beta\rho)\cdot q_t^g \cdot \beta\rho - c(q_t^g \cdot \beta\rho) - (P_t + \lambda P_t \frac{q_t^p}{\alpha\rho})\cdot q_t^p / \alpha\rho - c(q_t^p / \alpha\rho) \right)$$

$$\text{s.t.}\begin{cases} 0 \le q_t^g \le \bar{Q}^g \cdot U_t^g \\ q_t^g \le E_t - \underline{E} \\ 0 \le q_t^p \le \bar{Q}^p \cdot U_t^p \\ q_t^p \le \bar{E} - E_t \\ U_t^p \in \{0,1\}, U_t^g \in \{0,1\} \\ U_t^p + U_t^g \le 1 \\ E_{t+1} = \eta_t (E_t + q_t^p - q_t^g) \end{cases} \tag{20}$$

The parameters and constraints of PSH, market impact, and forecasted prices remain the same as cases 1-2 in Section 5.1. The above optimal results (unit commitment and economy dispatch (UCED) and optimal profit) in cases 1-2 are verified in AIMMS, a prescriptive analytics software. We achieve the same optimal results using both MILP methods and dynamic programming for the above two cases.

When the operating cost and efficiency loss are fixed values, the optimal profit and policy are most strongly related to the market impact λ. We confirm these findings by performing additional calculations, as briefly described next. We adjust the value λ from 0 to 0.2 increments of 0.01 and then re-run the cases in the software of AIMMS; the results do not differ materially from those obtained in case 1. It is, therefore, the market impact that will affect the optimal results. In the meanwhile, both cases also verify the relations of $E_{t+1}^{p*} \le E_{t+1}^{g*}$ for the scenario of price maker.

5.2 MISO Case Study

This section uses hourly time units as the power prices series $P = \{P_1, P_2, \ldots, P_T\}$ ($/MW) with 336 stages ($T = 336$) corresponding to two-week periods in MISO for the year 2020 as supplied. The first stage corresponding to the beginning of 06/28/2020 (the power prices are available at https://www.misoenergy.org/). Following the equation (2), we know that the updated prices at which the electricity merchant buys α_t units power from the market to increase her level (i.e., $\alpha_t > 0$) can be computed by $P_t + \lambda P_t \alpha_t / \alpha\rho$, and at which the electricity merchant sells α_t units power to the market to get revenue (i.e., $\alpha_t < 0$) can be expressed by $P_t + \lambda P_t \alpha_t \beta\rho$.

We assume the minimum and maximum storage capacity (upper reservoir) $\underline{E}$ and $\overline{E}$ are 2 and 20, respectively. Here, $\underline{E} > 0$ means the merchant cannot empty the storage, this is realistic in the power market for a PSH or battery owner. The generating and pumping rate constraint $\overline{Q}^p = 2$ and $\overline{Q}^g = 3$. The unit of measurement of storage can be interpreted as an appropriate MWH. The units of measurement of pumping and generating rate can be expressed as an appropriate MW)—both the pumping and the generating efficiency of $\alpha = \beta = 0.9$. The time $(\overline{E} - \underline{E})/\overline{Q}^g = 6$ hours units for the PSH to empty the upper reservoir, while $(\overline{E} - \underline{E})/\overline{Q}^p = 9$ hours units for the PSH to refill the upper reservoir correspond entirely approximately to the Taum Sauk pumped storage plant in Missouri USA. In this case, the maintenance and operating cost $c = 1$ (\$/MW). We assume $\rho = 1$ and $\eta = 1$.

When the power prices are fixed, the electricity merchant's revenue is determined mostly by the operating cost and efficiency in the trading. Here, we fix both the pumping and generating efficiencies, the operating cost, and focus on the market impact. We also assume the relationship between price and demand throughout the example can be obtained at any point in the time as the demand was varied. Usually, two weeks is an optimization cycle for the Taum Sauk in the power market.

The optimal policy obtained from the value functions (5) is shown in Fig 1 and Fig 2 with different initial energy in the storage, respectively.

Fig 1. Optimal Storage/SOC Change Curve with Market Impact (E1=2MWH)

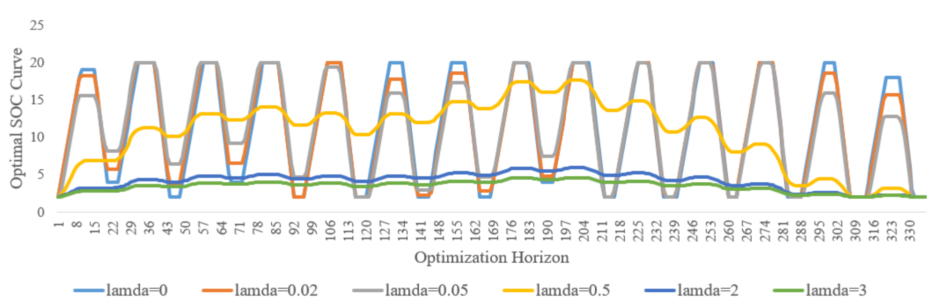
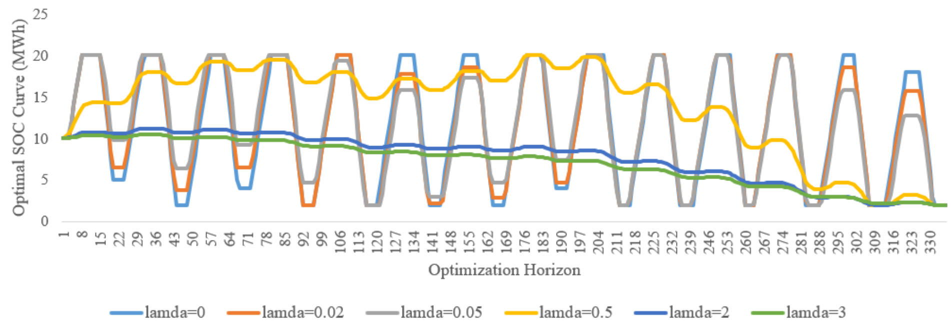


Fig 2. Optimal Storage/SOC Change Curve with Market Impact (E1=10MWH)



Figures 1-2 lead us to the following observations and conclusions. When the market impact is small, the merchant will adopt a similar strategy as the traditional policy (i.e., price-taker). We can also find the daily cycle of SOC according to the pattern of price every day. The merchant should buy power as much as possible at lower prices at midnight and sell as much as possible during the day at higher prices. However, when the market impact is large enough, we see that the navy-blue curve (λ=2) and green curve (λ=3) change very smoothly. To maximize profit, the electricity merchant needs to reduce the energy transition quantity (i.e., amount of energy that buys from the market or sells to the market) each period to lower the negative effect of market impact in trading. With the increase of market impact, the cost of buying will increase; however, the revenue will decrease. To reduce the negative effect of the market impact in trading, the electricity merchant should choose by lowering the power transition quantity at each period. Therefore, a profit-maximizing electricity merchant must perfectly balance the trade-off between the intensity of market impact and the power transition quantity. Next, we will show the relations between the expectation profit the market impact intuitively in Fig.3.

Fig 3. The Relations between Optimal Expectation Profit and Market Impact

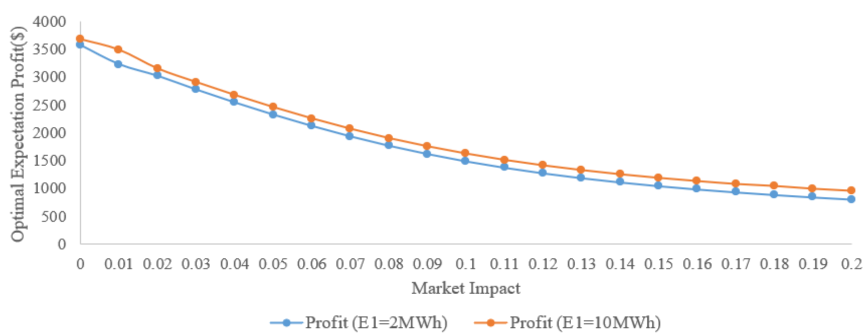


Fig 3 shows that comparing with the traditional study (i.e., price-taker merchant), if both price taker merchant and price maker merchant holds the same pumping and generating upper limits if the market impact is small or the electricity merchant's decisions have a weak market impact on prices, the merchant will get higher profit during trading. In this case, with the decrease in market impact, the cost will go down, and the revenue will rise, which benefits the merchant's profit. As the market impact increases, the cost of buying will increase; however, the revenue will decrease, which results in a lower profit.

The profit-maximizing merchant should reduce her market impact by balancing the intensity of market impact and the power transition quantity. In practice, the ratio of energy storage max capacity and the demand in the power market may be small, which means the market impact is low in trading. So, the merchant will adopt a similar optimal policy as the price taker; however, she may obtain less profit than theoretically maximum profit.

We also re-run the cases in the software of AIMMS by using one day period in MISO for 06/28/2020 as supplied. Once again, our previous conclusions are supported mainly. Therefore, it is reasonable to conclude that our results (i) are robust and representative of the broader set of parameters that we have tested, and (ii) confirm our analytical results and insights. We can draw the numerical simulations are in line with the conclusions made in Section 4 from our examination of Figures 1–3.

In summary, the scenario of price maker electricity merchants requires different trading strategies than a price taker. From the price-maker situation, results are novel and insightful—giving the electricity merchant an additional set of considerations when her trading decisions impact the market prices.

6 Conclusions and Future Research

Although the electricity trading policy has been extensively studied in the wholesale market and inventory management literature, optimal policy research from the respective price-maker whose trading decisions impact *market prices* has only started to gain attention recently. Our study is the first to model the problem as a Markov DP and derive the optimal policy structure for the slow storage to analyze electricity merchant management of managing a storage facility used for arbitrage and whose activities are sufficiently significant to have a market impact. We show that this optimal policy structure generalizes a classic result of Secomandi (2010) and differs significantly from typical threshold policies known to be optimal in the literature without considering the market impact. A focused yet thorough presentation required that we study only the merchant's objectives of maximizing profit.

For a price maker electricity merchant, in the presence of efficiency loss or operating cost, the optimal trading policy corresponding to the reference points/ functions at each decision time not only depend on the current energy availability in the storage and the given prices but also rely on its market price impact. The feasible SOC can be segmented into three regions by two optimal reference points/functions: buying-and-pumping, generating-and-selling, and do nothing (or idle/offline). However, suppose efficiency loss and operating costs are not considered. In that case, the feasible SOC can only be divided into two regions by one unique optimal reference point/function: buying-and-pumping and generating-and-selling. We obtain similar results and insights for electricity merchants after additionally incorporating the value of water/energy at the end of the optimization horizon into our notion of reward functions and value functions.

We identify the merchant's market impact as a critical driver of optimal policy design. If the electricity merchant's market impact is small on the prices, we will get similar results as the scenario price-taker. However, when both price-takers and price-makers are under the same generating and pumping limits offered to ISO, the most surprising finding is that market impact may lead to profit-reducing by increasing the cost of buying and decreasing sales revenue. If, besides, the market impact of electricity is high, then revenue can only partially offset the increased cost. In such a case, we find that the electricity merchant should lower the adverse effects of the market impact as much as possible by reducing the power transition quantity at each period to maximize the profit.

For analytical tractability, we assume in this paper that the market impact of electricity merchants follows a simple linear relation. Although we expect our paper's results to hold for other relationships, confirming this expectation is a worthy goal. This paper assumes that the price taker and price maker merchants have the same storage capacity and generating and pumping limits offered to ISO. How the results would change when the market impact is related to the storage capacity is another topic for future investigation. It seems likely that our paper's main structural results will also hold for other types of relation—when merchants choose to buy electricity, the market load will increase, leading to rising market prices; on the contrary, selling power by a price-marker merchant will increase the supply. Additionally, to keep the paper focused, we studied only the electricity merchant's objectives to maximize profit. It would also be necessary to understand the market impact of electricity merchants' decisions on social welfare and customer utility from ISO. Finally, this paper addressed only the case of the value of end water/energy in storage equals zero; it would be instructive to see how the results would change when the value of water/energy at the end of the optimization period is involved.

References

Bathurst, G. N., Strbac, G. (2003). Value of combining energy storage and wind in short-term energy and balancing markets. *Electric Power Systems Research*, 67 (1): 1-8.

Botterud, A., Levin, T., Koritarov, V. (2014). Pumped Storage Hydropower: Benefits for Grid Reliability and Integration of Variable Renewable Energy, Argonne National Laboratory Report. Available at https://publications.anl.gov/anlpubs/2014/12/106380.pdf

Brown, P. D., Peas, L. J. A., Matos, M. A. (2008). Optimization of pumped storage capacity in an isolated power system with large renewable penetration. *IEEE Transaction on Power Systems*, 23(2): 523-531.

Castronuovo, E. D., Lopes, J. A. P. (2004). On the optimization of the daily operation of a wind-hydro power plant. *IEEE Transactions on Power Systems*, 19(3): 1599-1606.

Charnes, A., Drèze, J., Miller, M. (1966). Decision and horizon rules for stochastic planning problems: A linear example. *Econometrica*, 34(2):307-330.

Chazarra, M., Perez-Diaz, J. I., Garcia-Gonzalez, J. (2018) Optimal Joint Energy and Secondary Regulation Reserve Hourly Scheduling of Variable Speed Pumped Storage Hydropower Plants. *IEEE Transactions on Power Systems*, 33(1):103-115.

Cheng, B., Powell, W. B. (2018). Co-Optimizing Battery Storage for the Frequency Regulation and Energy Arbitrage Using Multi-Scale Dynamic Programming. *IEEE Transactions on Smart Grid*, 9(3), 1997-2005.

Cruise, J., Flatley, L., Gibbens, R., Zachary, S. (2019). Control of Energy Storage with Market Impact: Lagrangian Approach and Horizons. *Operations Research*, 67(1): 1-9.

Deane, J.P., Gallachóir, B.P., McKeogh E.J. (2010). Techno-economic review of existing and new pumped hydro energy storage plant. *Renewable and Sustainable Energy Reviews*, 14: 1293-1302.

Dong, Z., Nelms, R., Muljadi, E., Tan, J., Gevorgian, V., Jacobson, M. (2018). Development of dynamic model of a ternary pumped storage hydropower plant, *2018 13th IEEE Conference on Industrial Electronics and Applications (ICIEA)*, 656-661, Wuhan, China.

Felix, B., Woll, O., Weber, C. (2012). Gas storage valuation under limited Market liquidity: An application in Germany. *European Journal Finance*, 19(7-8):715-733.

Harsha, P., Dahleh, M. (2015). Optimal management and sizing of energy storage under dynamic pricing for the efficient integration of renewable energy. *IEEE Transactions on Power Systems,* 30(3), 1164-1181.

Hashmi, M.U., Deka, D., Bušić, A. (2018). Limiting Energy Storage Cycles of Operation. *2018 IEEE Green Technologies Conference*. 71-74, Austin, Texas, USA.

Hashmi, M.U., Deka, D., Bušić, A., Pereira, L., Backhaus, S. (2019). Co-optimizing Energy Storage for Prosumers using Convex Relaxations. *2019 20th International Conference on Intelligent System Application to Power Systems (ISAP)*. 12.10-12.14, 10.1109/ISAP48318.2019.9065984, New Delhi, India.

Hashmi, M.U., Deka, D., Bušić, A., Pereira, L., Backhaus, S. (2020). Arbitrage with Power Factor Correction using Energy Storage. *IEEE Transactions on Power Systems,* 35(4):2693-2703.

Haslett, J. (1979). A Diffusion Model for the Storage of Solar Thermal Energy. *Journal of the Operational Research Society*, 38(5):433-438.

Huo, Y., Jiang, P., Zhu, Y., Feng, S., Wu, X. (2015). Optimal real-time scheduling of wind integrated power system presented with storage and wind forecast uncertainties. *Energies,* 8(2): 1080-1100.

Inderfurth, K., Kelle, P., Kleber, R. (2018). Inventory control in dual sourcing commodity procurement with price correlation. *Central European Journal of Operations Research*, 26: 93-119.

Jiang, D.R., Powell W.B. (2015). Optimal Hour-Ahead Bidding in the Real-Time Electricity Market with Battery Storage Using Approximate Dynamic Programming. *INFORMS Journal on Computing,* 27(3):525-543.

Huang, B., Chen, Y., Baldick, R. (2020). A Configuration Based Pumped Storage Hydro Model in MISO

Day-Ahead Market, https://arxiv.org/pdf/2009.04944.pdf

Kiedanski, D., Kofman, D., Horta, J., Menga, D. (2019). Strategy-proof local energy market with sequential stochastic decision process for battery control. *2019 IEEE Power & Energy Society Innovative Smart Grid Technologies Conference* (ISGT).

Kim, J. H., Powell, W. B. (2011). Optimal energy commitments with storage and intermittent supply. *Operation Research*, 59(6): 1347-1360.

Koritarov, V., Guzowski, L., Feltes J., Kazachkov, Y., Gong, B., Trouille, B., Donalek, P., Gevorgian, V. (2013). Modeling Ternary Pumped Storage Units, Argonne National Laboratory Report. Available at https://ceeesa.es.anl.gov/projects/psh/ANL_DIS-13_07_Modeling_Ternary_Units.pdf

Lee, J., Lee, E., Kim, J. (2020). Electric Vehicle Charging and Discharging Algorithm Based on Reinforcement Learning with Data-Driven Approach in Dynamic Pricing Scheme. *Energies*. 13(8), https://doi.org/10.3390/en13081950

Lifshitz, D., Weiss, G. (2015). Optimal energy management for grid-connected storage system. *Optimal Control Applications and Methods*, 36(4):447-462.

Löhndorf, N., Wozabal, D., Minner, S. (2013). Optimizing Trading Decisions for Hydro Storage Systems Using Approximate Dual Dynamic Programming. *Operations Research,* 61(4): 810-823.

Koritarov, V., Guzowski, L. Feltes, J.,Kazachkov, Y., Gong, B., Trouille, B., Donalek, P., Gevorgian, V. (2013). Modeling Ternary Pumped Storage Units, Argonne National Laboratory Report.

Available at https://ceeesa.es.anl.gov/projects/psh/ANL_DIS-13_07_Modeling_Ternary_Units.pdf

Mokrian, P., Stephen, M. (2006). A stochastic programming framework for the valuation of electricity storage. *26th USAEE/IAEE North American Conference*, Ann Arbor, MI

Midcontinent Independent System Operator (2020). MISO FORWARD 2020 focuses on the needs of Utilities of the Future. Available at https://www.misoenergy.org/about/media-center/miso-forward-2020-focuses-on-the-needs-of-utilities-of-the-future/

Nguyen, T.A., Byrne, R.H., Chalamala, B.R., Gyuk, I. (2018). Maximizing the Revenue of Energy Storage Systems in Market Areas Considering Nonlinear Storage Efficiencies. 2018 International Symposium on Power Electronics, Electrical Drives, Automation and Motion, 55-62, Italy.

Porteus, E.L. (2002). Foundations of Stochastic Inventory Theory. Stanford University Press, Palo Alto, CA.

Puterman, M. L. (1994). Markov Decision Processes: Discrete Stochastic Dynamic Programming. John Wiley & Sons Press, Hoboken, NJ.

Powell, W.B., George, A., Simão, H., Scott, W., Lamont, A., Stewart, J. (2011). SMART: A Stochastic Multiscale Model for the Analysis of Energy Resources, Technology, and Policy. *INFORMS Journal on Computing*, 24(4): 665-682.

Powell, W.B., Meisel, S. (2016). Tutorial on Stochastic Optimization in Energy---Part I: Modeling and Policies. *IEEE Transactions on Power Systems*, 31(2): 1459-1467.

Powell, W.B., Meisel, S. (2016). Tutorial on Stochastic Optimization in Energy---Part II: An Energy Storage Illustration. *IEEE Transactions on Power Systems*, 31(2): 1468-1475.

Qi, W., Liang, Y., Shen Z.M. (2015). Planning Energy Storage and Transmission for Wind Energy Generation. *Operations Research*, 63(6): 1280-1293.

Richmond, N., Jacko, P., Makowski, A. M (2014). Optimal Planning of Slow-Ramping Power Production in Energy Systems with Renewables Forecasts and Limited Storage, *2014 International Conference on Probabilistic Methods Applied to Power Systems (PMAPS)*, 7-10 July, Durham, UK.

Roberts, B. (2009). Capturing grid power. *IEEE Power and Energy Magazine*, 7(4): 32-41.

Secomandi, N. (2010). Optimal commodity trading with a capacitated storage asset. *Management Science*, 56(3): 449-467.

Steeger, G., Rebennack, S. (2015) Strategic bidding for multiple price-maker hydroelectric producers. *IIE Transactions*, 47(9):1013-1031

Steffen, B., Weber, C. (2016). Optimal operation of pumped-hydro storage plants with continuous time-varying power prices. *European Journal of Operation Research,* 252(1): 308-321.

Water Power Technologies Office (2020). Office of Energy Efficiency & Renewable Energy, Enegy.Gov. Available at https://www.energy.gov/eere/water/pumped-storage-hydropower.

Wei, L., Guan, Y. (2014) Optimal Control of Plug-In Hybrid Electric Vehicles with Market Impact and Risk Attitude. *Transportation Science*, 48(4): 467-482.

Wiser, R., Bolinger, M. (2015). Wind technologies market report. Tech. Rep., U.S. Department of Energy. Available at https://energy.gov/sites/prod/files/2016/08/f33/2015-Wind-Technolo-gies-Market-Report-08162016.pdf

Williams, J. C., Wright, B. D. (1991). Storage and Commodity Markets. Cambridge University Press, Cambridge, UK.

Yeh, W.G. (1985). Reservoir Management and Operations Models: A State-of-the-Art Review. *Water Resources Research*, 21(12):1797-1818.

Zhan, S., Hou, P., Enevoldsen, P., Yang, G., Zhu, J., Eichman, J., Jacobson, M. Z. (2020) Co-optimized trading of hybrid wind power plant with retired EV batteries in energy and reserve markets under uncertainties. *International Journal of Electrical Power & Energy Systems*, https://doi.org/10.1016/j.ijepes.2019.105631

Zhang, L., Wirth, A. (2010). Wind energy management with battery storage, *Journal of Operational Research Society*, 61(10): 1510-1522.

Zhou, Y., Scheller-Wolf, A., Secomandi, N., Smith, S. (2019). Managing Wind-Based Electricity Generation in the Presence of Storage and Transmission Capacity. *Production and operations management*, 28(4): 970-989.

Zhou, Y., Scheller-Wolf, A., Secomandi, N., Smith, S. (2016). Electricity trading and negative prices: storage vs. disposal. *Management Science*, 62(3): 880-898.