Optimal Offering Strategy of GenCo with Joint Participation in FTR Auction and Day-Ahead Market Considering Virtual Bidding

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***Abstract*— Nodal price separations in Day-Ahead (DA) market caused by transmission congestion create congestion charges/surplus that are reallocated to the market participants through the financial transmission right (FTR) auction. From a market participant's viewpoint, these two markets are interrelated because the revenue of market participant in FTR auction is determined based on the day-ahead locational marginal prices. Furthermore, virtual transactions which are designed to improve price convergence between the day-ahead and real-time markets can directly impact day-ahead prices. This impact through virtual transactions may be utilized by a market participant to increase its FTR value and improve its overall strategy in participating in both FTR auction and day-ahead market. To this end, this paper is the first attempt to reveal this tactic in literature and proposes an offer strategy framework for a price-maker generating company participating in both FTR auction and day-ahead market with the consideration of virtual bidding. First, the possibility of FTR value manipulation is conceptually demonstrated by placing virtual bids in the day-ahead market. Second, a two-stage bi-level offering strategy model is formulated for strategic GenCos, which is further converted to a single-level optimization problem by using Karush-Kuhn-Tucker conditions and strong duality theory. Numerical tests on a 5-bus system and IEEE reliability test system (RTS) demonstrate the effectiveness and applicability of the proposed approach.**

***Index Terms*—Bidding, day-Ahead market, FTR auction, manipulation, offering strategy, virtual bidding.**

NOMENCLATURE

A. Abbreviations

DA	Day-Ahead
RT	Real-Time
MP	Market Participants
FTR	Financial Transmission Right
SDT	Strong Duality Theory
LMP	Locational Marginal Price
GenCo	Generating Company
KKT	Karush-Kuhn-Tucker
DEC	Decrement Bid
INC	Increment Bid
UL-FTR	Upper Level of the 1st stage model [bidding in FTR auction]
LL-FTR	Lower Level of the 1st stage model [bidding in FTR auction]
UL-DA	Upper Level of the 2nd stage model [bidding in DA Market]
LL-DA	Lower Level of the 2nd stage model [bidding in DA Market]

B. Indices and Sets

t	Index for time periods
v	Index for virtual bids
i	Index for strategic generating units
c	Index for FTR seller
e	Index for strategic FTR buyer
j	Index for other generating units
f	Index for other FTR buyers
b	Index for generation blocks
d	Index for demands
k	Index for demand blocks
l	Index for lines
n	Index for buses
m	Index for FTR paths
$sink, source$	Index for Sink and Source buses
N_{pur}, N_{sell}	Set of sellers and buyers in the FTR auction
Ω_1^{FTR}	Set of decision variables in UL of FTR auction model: $\Omega_1^{FTR} = \{FTR_e^{Sbid}, \rho_e^s\}$
Ω_2^{FTR}	Set of decision variables in LL of FTR auction model: $\Omega_2^{FTR} = \{FTR_e^S,\ FTR_f, FTR_c\}$.
Ω_1^{DA}	Set of decision variables in UL of DA market model: $\Omega_1^{DA} = \{P_{tib}^{Sbid}, \alpha_{tib}^S, V_{tv}^{bidG},\ \alpha_{tv}^{bidG}, V_{tv}^{bidD}, \alpha_{tv}^{bidD}, Ug_{tv}, Ud_{tv}\}$
Ω_2^{DA}	Set of decision variables in LL of DA market model: $\Omega_2^{DA} = \{P_{tib}^S, P_{tjb}^g, P_{tdk}^d, V_{tv}^{DAg}, V_{tv}^{DAd}\}$

C. Parameters

$\overline{FTR}_e^S$	Upper limit of FTR for strategic FTR buyer *e*
$\overline{FTR}_f$	Upper limit of FTR for other FTR buyer *f*
$\overline{FTR}_c$	Upper limit of FTR for FTR seller *c*
σ_c	Offer price of FTR seller *c*
ρ_f	Bid price of FTR buyer *f*
H_{lm}	Sensitivity of line *l* to the FTR MW in path *m*
λ_{tn}^{RT}	Real-Time locational marginal price at time t and bus n
λ_{tib}^S	Marginal cost of unit i of the strategic generator at time t
$\bar{P}_{tib}^S$	Upper power bound of unit i of the strategic generator at time t

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$\bar{P}_{tjb}^{G}$	Upper power bound of unit j of other generators at time t
$\bar{P}_{tdk}^{D}$	Upper power limit of demand d at time t
λ_{tjb}^{g}	Marginal cost of unit j of other generators at time t
λ_{tdk}^{d}	Marginal utility of demand d of the at time t
V_{tv}^{budget}	Upper quantity limit of virtual bid v at time t
$\bar{F}_{l}$	Line l Capacity
$PTDF_{nl}$	Power Transfer Distribution Factor
R_{i}^{UP}, R_{i}^{LO}	Ramp-up and ramp-down limits for strategic unit i

D. Variables

FTR_{e}^{Sbid}	Bid quantity for FTR bidder e
FTR_{e}^{s}	Cleared quantity for FTR bidder e
FTR_{c}	Cleared FTR for seller c
FTR_{f}	Cleared FTR for buyer f
ρ_{e}^{s}	Bid price for FTR bidder e
LF_{l}	Power flow in line l in FTR auction
F_{l}	Power flow in line l in DA market
P_{tib}^{Sbid}	Bid power of the strategic generating unit i at time t
P_{tib}^{S}	Cleared power of the strategic generating unit i at time t
α_{tib}^{S}	Bid price of the strategic generating unit i at time t
α_{tv}^{bidG}	Bid price of virtual generation v at time t
α_{tv}^{bidD}	Bid price of virtual demand v at time t
V_{tv}^{bidG}	Bid quantity of virtual generation v at time t
V_{tv}^{bidD}	Bid quantity of virtual demand v at time t
V_{tv}^{DAg}	Cleared quantity of virtual generation v at time t
V_{tv}^{DAd}	Cleared quantity of virtual demand v at time t
P_{tjb}^{G}	Cleared power generation of unit j of other generators at time t
P_{tdk}^{D}	Cleared power for load d at time t
Ug_{ti}, Ud_{ti}	Binary variables determine the virtual generation or virtual demand
Θ	Objective function value of the second stage problem

E. Dual Variables

MCP_{m}	FTR auction price for FTR in path m
τ	Lagrangian coefficients of the FTR quantity limitations in the LL problem of the 1st stage optimization model
ξ	Lagrangian coefficients of the line capacity constraints in the LL problem of the 1st stage optimization model
LMP_{tn}	Locational Marginal Price at time t and bus n in the DA market
μ	Lagrangian coefficients of the generation and demand limitation in the LL problem of the 2nd stage optimization model
ϑ	Lagrangian coefficients of the line capacity constraints in the LL problem of the 2nd stage optimization model
γ, u, ω	Binary variables needed to linearize the complementary constraints

I. Introduction

After restructuring of the power industry, market participants (MPs) may be exposed to high and unpredictable congestion charges due to locational marginal pricing (LMP) in wholesale electricity markets. To protect the MPs from the congestion price uncertainty and to provide the fair approach of allocating the leftover funds, independent system operators (ISOs) hold an independent auction called a Financial Transmission Right (FTR) auction in which the FTRs values are determined based on the day-ahead (DA) LMP differences between the beginning nodes ($source$) and the end nodes ($sink$) of the FTRs paths. This financial instrument provides an opportunity to the MP to hedge risk and at the same time creates a possibility to manipulate the wholesale market prices to maximize its profit profile. The crucial issue is how a strategic MP should build its offering strategy in both the wholesale market and the FTR auction, which is addressed in this paper.

FTRs, known as point-to-point FTRs, are financial instruments that allow the market participants to obtain an annual or monthly share of surplus congestion charges collected by ISO. These surplus charges result from the disparity of LMPs in congested areas, which leads to more fund collected from demands by ISO than payment to generation suppliers [1]. These rights can be categorized as “obligation” and “option”. The major distinction between these FTRs arises when the LMP difference between the sink (withdrawal) and source (injection) buses is negative, which makes the FTR obligation a liability, yet FTR option would never be a liability [2]. A generalized FTR auction model was reported in [3], where market players were allowed to purchase and sell FTR through the auction. Moreover, implementation of obligation FTR auction in the PJM market was presented in [4]. Finding the optimal bidding strategy for market players in the FTR auction has been studied in the literature. The bi-level optimization models were utilized in [2] and [5] to maximize the MPs’ FTR payoffs in the FTR auction. Using the conjectured price influence function, [6] evaluated the strategic bidder’s market power in the FTR auction. A game theoretic model was used in [7] to develop the optimal bidding for generators, and [8] modeled the FTRs based on the equilibrium condition.

In terms of bidding strategy problems, many works on designing the bidding decision designs of different MPs in the DA and Real-Time (RT) markets can be found in the literature. For instance, the bidding decisions of price-taker MPs, whose actions cannot change the market outcomes, were studied in [9]–[15], while the bidding strategy problem for the price-maker MPs, whose behaviors impact the market results, were investigated in [16]–[21]. These works employed different approaches to find the optimal bidding decisions in electricity markets, primarily DA markets, without considering the effect of FTR auction outcome on their strategies. Decision-making process of different MPs participating in multiple markets was investigated in [22]–[24]; however, very few works provided the methodologies for MPs to optimize their behaviors in FTR auction and DA markets [25]–

[26].

Virtual transactions (also known as virtual bids or convergence bids) are practical instruments used to compensate for the gaps between the LMPs in DA and RT markets. According to the PJM report [27, 28], financial players can trade virtual bids as increment offers (INC) or decrement bids (DEC) in the DA market without any intention of generating or consuming the real power in the RT market. The merits and demerits of virtual bids participating in the DA market, were discussed in [29]–[31]. Virtual bids were employed as flexible resources in [31] in four distinct two-settlement market clearing models to improve the DA scheduling of generating units. A three-stage equilibrium model was introduced in [32] to evaluate the manipulation in three sequential markets, considering the demand and congestion uncertainties. Moreover, numerical simulation on a two-bus system showed that the DA price manipulation by virtual bids, thus taking advantage of FTR positions, was achieved when all players engaged in the Cournot game in the DA market. A Mixed Density Network (MDN) was introduced in [33] to forecast the LMP difference between RT and DA markets, and it presented a data-driven algorithmic bidding strategy for virtual bids in the DA market. [34] evaluated the strategy of a photovoltaic producer using the virtual bids and stochastic optimization. The optimal bidding decision design for a virtual bidder in the DA market, considering the risk of profit volatility, was presented in [35]. In addition to all the aforementioned benefits and applications, virtual bids are able to increase the value of FTR in an FTR auction because they can be submitted as generations or loads at specific locations [36]. This feature of virtual bid provides a potential opportunity for the MPs to enhance their bidding strategy designs, which has not been studied in existing works.

Therefore, this paper proposes a two-stage bi-level optimization model for designing a joint offering strategy for a strategic generation company (GenCo) which participates in both FTR auction and DA market and is capable of submitting virtual bids in the DA market. Strategic GenCo tends to maximize its payoff in the FTR auction which is modeled in the lower-level (LL) of the first stage problem. Furthermore, the revenue of the GenCo in FTR auction is dependent on the LMP difference between the sink and source buses, which are transferred from the second stage problem (DA market model). In addition, the strategic decision-making process of this MP in the DA market is modeled in the second stage problem, in which the upper-level (UL) subproblem models the GenCo's profit maximization problem. The only relation between the first stage and second stage problems is the DA LMP, which appears in the first stage objective function, meaning that this problem can be written as a single stage bi-level problem. Finally, employing KKT optimality conditions and strong duality theory (SDT), this problem is transformed into its equivalent single-stage, single-level problem.

In comparison with [35] and [37] which designed the MP's bids for only DA market, the proposed work investigates MP's offering strategy problem of joint participation in two interrelated markets, DA market and FTR auction. Specifically, the proposed work reveals strategies that can influence FTR value through manipulating DA market. It is a topic rarely studied in the literature, and yet bears high value in actual market practices. [25] studied the joint offering problem from the perspective of a physical MP, while in the proposed work, physical MP with virtual bidding capability is the focused subject because virtual bidding is a powerful tool for MPs to manipulate DA market which subsequently affects the FTR value. In addition, [25] studied DA market bidding strategy with the consideration of cleared FTR auction results. FTR auction is not modeled, and therefore in essence [25] is a DA market bidding problem, which significantly limits its offering flexibility and profit maximization ability. In contrast, the proposed work treats the MP as a "price-maker" player in both FTR and DA markets, to maximize the bidding opportunities. FTR auction is modeled to evaluate the participation strategy in FTR auction. Therefore, the contributions of this paper can be summarized as follows.

1) Reveal the possibility of FTR value manipulation by submitting virtual bids in the DA market, which allows the MPs to design their strategies with the consideration of broader decision-making horizon.

2) Provide a systematic framework to investigate the possibility of such manipulation with a novel two-stage bi-level joint offering strategy model for strategic GenCo participating in both FTR auction and DA market. In addition to deciding on the physical generation of the GenCo, the proposed model allows this MP to employ virtual bids to optimize its profit considering its flexibility of being either load or generation at different nodes of the network, which affects the FTR value and the MP's offering strategy overall performance in both markets.

3) Demonstrate the effect of FTR auction outcomes on the offering decision of the strategic GenCo in the DA market by studying the behavior of the GenCo engaging separately and jointly in both markets. Furthermore, several case studies are designed to compare with existing methods and illustrate the efficacy of the proposed model in developing an improved offering strategy and demonstrate the opportunity for a strategic MP to obtain higher total profit than the combined profits in FTR auction and DA market using individually developed strategies in the literature.

The rest of the paper is organized as follows. Section II demonstrates the possibility of FTR value manipulation by submitting virtual bids in the DA market. The proposed joint offering strategy model is briefly described in Section III, and the mathematical formulation is fully detailed in Section IV. Illustrative example and case study results are discussed in Section V and Section VI, respectively. Lastly, Section VII concludes the paper.

II. Manipulation of FTR Value by Virtual Bidding

A. Virtual Bidding

Virtual transactions that can be submitted into the market in form of either demand (DEC) or generation (INC), are able to change the market prices. A distinct feature of virtual bid is that the amount of power bought (sold) by the virtual bidder in the DA market is exactly compensated by a sale (purchase) of the same amount of power in the RT market, such that the net amount of power traded in these markets is zero. For instance, if a virtual bidder expects a lower DA LMP than RT LMP at the specific bus, he/she could submit the virtual demand (DEC) in that bus for the same time-period. If the submitted DEC is cleared, the virtual bidder pays the DA LMP on all purchased power and receives the RT LMP for the same amount of power sold in the RT market. Therefore, the virtual bidder's profit is equal to the DA/RT LMP difference multiplied by the amount of cleared virtual power.

Simply, the value of a virtual bid is estimated based on the DA/RT LMP differences. To illustrate this point, the modified version of a five-bus test system defined in [37] is employed here (Fig. 1). There are 5 generators, 4 loads, 6 transmission lines in this system. All offer (bid) quantities and generators' (loads') prices, and the transmission line capacities are depicted in Fig. 1 beside each element. Assume that a virtual bidder intends to submit VB amount of DECs at bus B. Considering the fixed forecasted RT LMP, the value of the virtual bid ($\lambda^{RT}-\lambda^{DA}$) decreases because the DA LMP increases, when the amount of DECs increases at this bus (Fig. 2). The more cleared amount of DEC, the more profit the virtual bidder can make. However, the price spread between DA LMP and RT LMP declines when the amount of cleared DEC increases, thus the virtual bidder's profit decreases. As is seen in Fig. 2, the virtual bidder can maximize the profit ($294.64) by placing 9MW DECs at bus B.

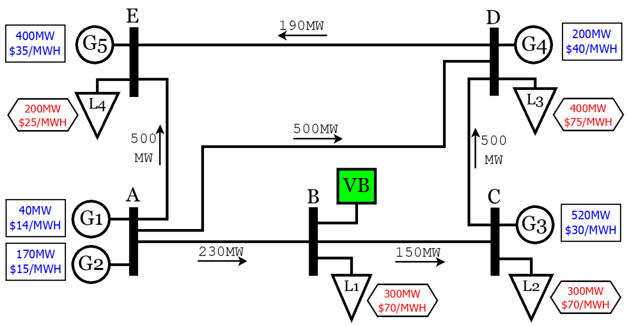

Fig. 1. Five-bus test system

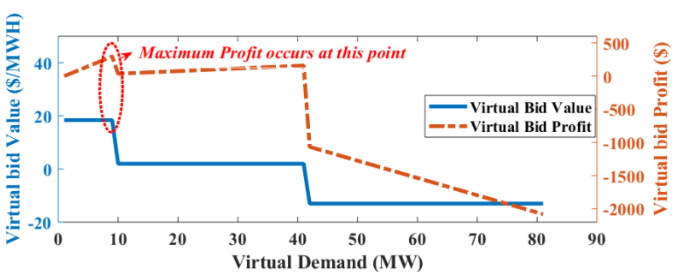

Fig. 2. Virtual demand value and virtual profit of trader in 5-bus system

B. Manipulation of FTR Value by Virtual Bids

By contrast, as it was comprehensively explained in [7], the FTR allows the MPs to hedge against the congestion risk; and its value is equal to the LMP difference between the *sink* and *source* buses. For example, if an MP owns F MW FTR from bus "E" (*source*) to bus "B" (*sink*), its revenue would be equal to $(\lambda_E^{DA}-\lambda_B^{DA})\times F$. As this instrument's value is calculated based on the DA LMP, there is an opportunity for an FTR holder to manipulate the DA LMP, thus maximizing its payoff. Moreover, as it was shown in the Section II-A, an MP can change the DA LMP without the obligation of generating or consuming any physical power. Therefore, placing virtual bids at specific buses in the system can worsen the line congestion in the DA market and increase the DA LMP difference between the sink and source buses, then it provides more FTR profit for the MP. To illustrate this point, the previous example is extended by assuming that the MP holds an FTR from bus "E" to bus "B". As is shown in Fig. 3, DA LMP at bus "B" increases when the more virtual demand (DEC) is cleared at this bus; therefore, the FTR value ($\lambda_E^{DA}-\lambda_B^{DA}$) increases, and MP makes more FTR profit.

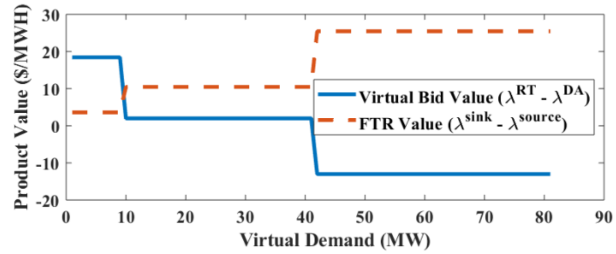

Fig. 3. Virtual demand value and FTR value profiles by placing DECs at bus "B" of 5-bus system

Fig. 4 shows the total profit from the FTR and virtual trades. As shown, the maximum value of total profit is $994.4 that happens when 41MW virtual demand is cleared in the DA market. Although the MP loses a small amount of money in the DA market by submitting more virtual demand, its total profit is maximized by making more FTR profit. In other words, the ability of the MP to increase the value of FTR, incentivizes him/her to place more DECs at the sink bus and optimize the total payoff.

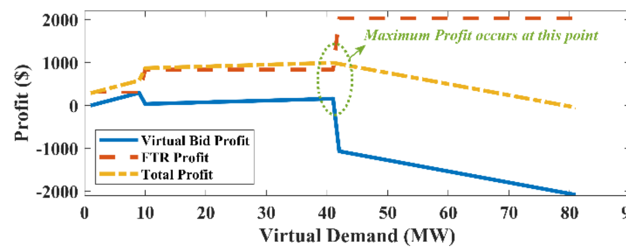

Fig. 4. Virtual profit, FTR profit and total profit profiles of MP by placing DECs at bus "B" of 5-bus system

C. Role of Virtual Bidding in the Joint DA-FTR Bidding Problem

While virtual bidding is designed to provide a market mechanism to drive the convergence of DA and RT markets, it can be utilized by strategic MPs to manipulate DA LMP and, through the FTR auction settlement, the FTR value of FTRs owned by the strategic MPs. Therefore, virtual bids can be used as a tool by the strategic MPs to maximize their collective profits in participating both DA markets and FTR auction market by means of influencing the day-ahead market prices and FTR value. In the proposed model, the MP's ultimate goal is to maximize the total profit by strategically participating in the FTR auction and DA market. More specifically, the strategic MP submits the virtual bids alongside the physical generation offers into the DA market to efficiently manipulate the market prices for its own interest.

III. Proposed Joint Offering Strategy Model Framework

A. Model Assumptions

The main assumption of the proposed model is described here for clarity.

1) Transmission network is modeled using DC power flow to be consistent with contemporary market practices. Power Transfer Distribution Factor (PTDF) has been used to calculate the line flows.
2) Unlike [25], MP is considered a price maker player in both FTR auction and DA market to investigate the influence of MP's strategies in both markets simultaneously.
3) To align with real-world practices [41,43], MP is assumed to participate in a one-round FTR auction to acquire the FTRs.

The FTR auction is held one month prior to the DA markets which is executed for 24 hourly periods.

B. Model Description

FTRs provide MPs a valuable way to protect them against price uncertainties due to congestions. Payments to FTR holders are determined by DA LMP settled in DA market. The DA LMP difference between *source* and *sink* buses of a FTR is dependent to the MPs' offering strategies and settlement of the DA market. Therefore, the MP's offering strategy design procedures in FTR auction and DA market are strongly correlated and need to be studied together. As a result, players in the FTR and DA markets may choose to develop bidding strategies that maximize the combined payoffs in the two markets. To this end, this paper proposes a two-stage two-level model to achieve that.

Fig. 5 represents the time sequence of power markets. As a part of forward markets, monthly FTR auction is held a month prior to the DA market [2], and an MP requires the forecasted DA LMP to design its FTR bidding strategy. This study aims to design a bidding strategy for a price-maker MP participating in both the monthly FTR auction and Day-ahead market.

Fig. 5. Time sequence of different electricity markets.

Fig. 6 presents the decision graph of the proposed model. As shown, the total profit of the strategic MP comprises the FTR profit and the profit from the physical generation and virtual bids in the DA markets. Interdependency of these two markets, from the MP's viewpoint, comes from the DA LMP of the *sink* and *source* buses, which are determined after the DA market clearing process and are required to compute the revenue of MP from its FTR position.

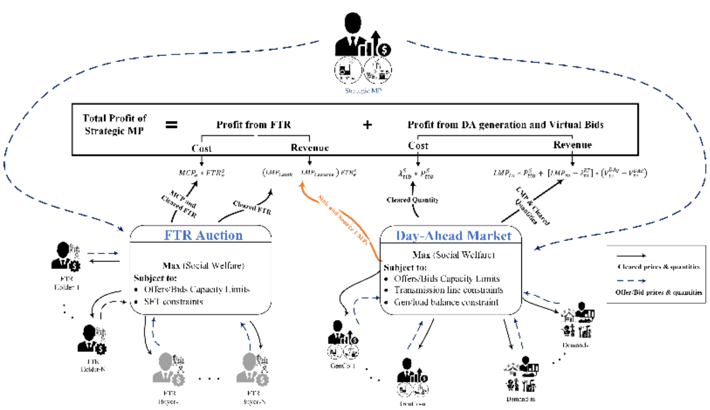

Fig. 6. Bidding Strategy of strategic MP in both FTR and DA markets

A DA market model can be used by a strategic MP to observe the market's reaction to the imposed strategies. Furthermore, an FTR auction model is needed to derive the market price in the FTR auction, which is required to calculate the FTR cost [7]. As a result, in this paper, a two-stage bi-level optimization model is proposed to capture the strategic MP's offering strategy in FTR auction and DA market. As shown in Fig. 7, the first stage problem is a bi-level optimization model that maximizes the MP's profit in the FTR auction in the UL subproblem, and it models the FTR auction clearing process in the LL subproblem. At this stage, the DA LMPs required to calculate the FTR revenue, come from the second stage. At the second stage, another bi-level optimization problem, that models the maximization problem at the UL and the DA clearing procedure at the LL subproblems, is formulated. The detailed description of the proposed model is fully explained in Section IV.

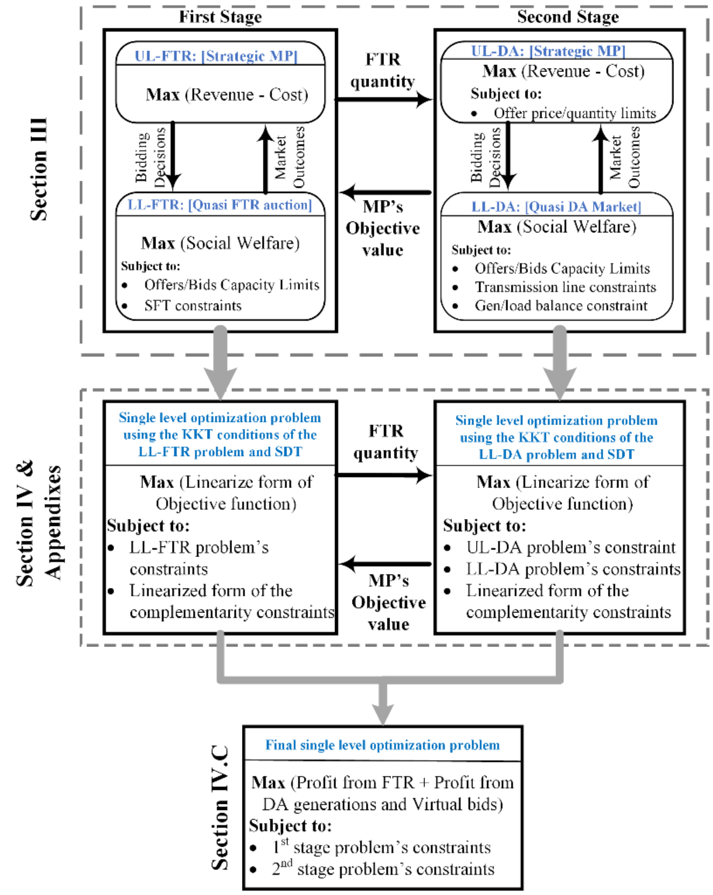

Fig. 7. Proposed two-stage bi-level optimization model and the solution methodology

IV. Mathematical Formulation

To facilitate the explanation of how the model is constructed, each of the stages is modeled separately and then the required exchanged information between the stages is specified and the final model is presented. Therefore, the first stage two-level model, which represents the bidding strategy model in FTR auction, is explained at the first step, then the second stage two-level model is discussed. Lastly, the final model is formed according to the transferred data between these stages.

A. First Stage: Bidding strategy model in FTR Auction

The first stage of the proposed model tries to maximize the MP's profit participating in the FTR auction. To consider the influence of MP's bids on the FTR auction price, two-level optimization model is formed as follows:

Upper-Level

$$\underset{\Omega_1^{FTR}}{\text{Min.}} \sum_t \left(MCP_e FTR_e^s - \left(LMP_{t,sink} - LMP_{t,source} \right) FTR_e^s \right) \tag{1a}$$

s.t:

$$0 \le FTR_e^{Sbid} \le \overline{FTR}_e^S \tag{1b}$$

$$\rho_e^s \geq 0 \tag{1c}$$

Lower-Level

$FTR_e^s, MCP_e \in \arg\{$

$$\underset{\Omega_2^{FTR}}{\text{Min.}} \sum_{c \in N_{sell}} \sigma_c FTR_c - \sum_e \rho_e^s FTR_e^s - \sum_{f \in \{N_{pur}-e\}} \rho_f FTR_f \tag{1d}$$

s.t:

$$0 \leq FTR_e^s \leq FTR_e^{Sbid} : \underline{\tau}_e^S, \overline{\tau}_e^S, \ \forall e \tag{1e}$$

$$0 \leq FTR_f \leq \overline{FTR}_f : \underline{\tau}_f, \overline{\tau}_f, \forall f \in \{N_{pur} - e\} \tag{1f}$$

$$0 \leq FTR_c \leq \overline{FTR}_c : \underline{\tau}_c, \overline{\tau}_c, \forall c \in N_{sell} \tag{1g}$$

$$-\overline{F}_l \leq LF_l^{ex} + LF_l \leq \overline{F}_l : \underline{\xi}_l, \overline{\xi}_l \quad \forall l \tag{1h}$$

$$LF_l = \sum_m H_{lm} FTR_m, \forall l, m \in \{N_{sell}, N_{pur}\} \tag{1i}$$

$$MCP_m = \sum_l H_{lm}(\overline{\xi}_l - \underline{\xi}_l), \forall m \in \{N_{sell}, N_{pur}\} \tag{1j}$$

$$H_{lm} = PTDF_{l,source} - PTDF_{l,sink} \tag{1k}$$

The objective of the MP is to minimize the negative of profit in FTR auction using (1a) that models the FTR cost in the first term ($MCP_e FTR_e^s$) and FTR revenue in the second term ($(LMP_{t,sink} - LMP_{t,source})\, FTR_e^s$). FTR unit price ($MCP_e$) comes from the FTR auction clearing procedure that is modeled at the LL subproblem ((1d)–(1k)). Additionally, FTR revenue requires the second stage information (DA LMP difference between sink and source buses) to be calculated. In this model $\Omega_1^{FTR} = \{FTR_e^{Sbid}, \rho_e^s\}$ is the MP's set of decision variables in the UL subproblem. FTR bid quantity is bounded by (1b) and (1c), forcing the FTR bid price to be positive. FTR auction model seeks to minimize the minus social welfare [1] regarding the simultaneous feasibility test (SFT) constraints. LL decision variables are represented by $\Omega_2^{FTR} = \{FTR_e^s, FTR_f, FTR_c\}$. Cleared FTR quantities are bound by their maximum and minimum FTR bids or offers in (1e)–(1g). Constraint (1h) limits the line flows calculated by (1i) to the transmission line capacities. LF_l^{ex} in (1h) denotes the line flows caused by the existing FTRs contracted in the secondary market [1, 3]. Employing the Lagrangian coefficients of (1h), FTR auction price can be determined by (1j). Note that shift factor versus FTR bids/offers (H_{lm}) is required to calculate the line flows and FTR auction price in this model. This parameter can be obtained by subtracting the PTDF of the line l vs. the $source$ from that of the $sink$ buses (1k). The FTR auctioneer takes ρ_e^s as a parameter, meaning the LL subproblem is linear and convex, thus, it is replaced by KKT conditions. Therefore, the Model (1) is written as a single level optimization problem known as MPEC which is detailed in APPENDIX A.

B. Second Stage: Bidding strategy model in DA market

As explained, FTR revenue is calculated using the DA LMPs at $sink$ and $source$ buses. It is assumed that the MP is a price-maker player in the DA market, thus it is needed to model the MP's bidding strategy decision-making problem in the DA market and study the effects of its offers (quantity and price) on the DA market clearing outcomes. Furthermore, the strategic MP submits the virtual bids from different locations in the DA market, which empowers the MP to change the DA LMPs for its own interest. Therefore, the second stage of the proposed model represents the bidding strategy problem of the MP and aims to maximize the MP's payoff in the DA market as follows:

Upper-Level

$$\underset{\Omega_1^{DA}}{\text{Min.}} \sum_{t(i \in \psi_n)b} [\lambda_{tib}^S - LMP_{tn}]\, P_{tib}^S - \sum_{t(v \in \psi_n)} [LMP_{tn} - \lambda_{tn}^{RT}]\,(V_{tv}^{DAg} - V_{tv}^{DAd}) \tag{2a}$$

s.t:

$$\sum_b P_{(t+1)ib}^s - \sum_b P_{tib}^s \leq R_i^{UP}, \quad \forall t, i \tag{2b}$$

$$\sum_b P_{tib}^s - \sum_b P_{(t+1)ib}^s \leq R_i^{LO}, \quad \forall t, i \tag{2c}$$

$$0 \leq P_{tib}^{Sbid} \leq \overline{P}_{tib}^S, \quad \forall t, i, b \tag{2d}$$

$$0 \leq V_{tv}^{bidG} \leq V_{tv}^{budget} Ug_{tv}, \quad \forall t, v \tag{2e}$$

$$0 \leq V_{tv}^{bidD} \leq V_{tv}^{budget} Ud_{tv}, \quad \forall t, v \tag{2f}$$

$$Ug_{tv} + Ud_{tv} \leq 1, \quad \forall t, v \tag{2g}$$

$$\alpha_{tib}^S \geq 0, \alpha_{tv}^{bidG} \geq 0, \alpha_{tv}^{bidD} \geq 0 \tag{2h}$$

Lower-Level

$P_{tib}^S, V_{tv}^{bidG}, V_{tv}^{bidD} \in \arg\{$

$$\underset{\Omega_2^{DA}}{\text{Min.}} \sum_{tib} \alpha_{tib}^S P_{tib}^S + \sum_{tjb} \lambda_{tjb}^g P_{tjb}^g - \sum_{tdk} \lambda_{tdk}^d P_{tdk}^d + \sum_{tv} (\alpha_{tv}^{bidG} V_{tv}^{DAg} - \alpha_{tv}^{bidD} V_{tv}^{DAd}) \tag{2i}$$

s.t:

$$\sum_{v \in \psi_n} (V_{tv}^{DAg} - V_{tv}^{DAd}) + \sum_{(i \in \psi_n)b} P_{tib}^S + \sum_{(j \in \psi_n)b} P_{tjb}^g - \sum_{(d \in \psi_n)k} P_{tdk}^d = inj_{tn} : LMP_{tn}, \quad \forall t, n \tag{2j}$$

$$0 \leq P_{tib}^s \leq P_{tib}^{Sbid} : \underline{\mu}_{tib}^S, \overline{\mu}_{tib}^S, \ \forall t, i, b \tag{2k}$$

$$0 \leq P_{tjb}^g \leq \overline{P}_{tjb}^G : \underline{\mu}_{tjb}^G, \overline{\mu}_{tjb}^G, \ \forall t, j, b \tag{2l}$$

$$0 \leq P_{tdk}^d \leq \overline{P}_{tdk}^D : \underline{\mu}_{tdk}^D, \overline{\mu}_{tdk}^D, \ \forall t, d, k \tag{2m}$$

$$0 \leq V_{tv}^{DAg} \leq V_{tv}^{bidG} : \underline{\mu}_{tv}^{Vg}, \overline{\mu}_{tv}^{Vg}, \ \forall t, v \tag{2n}$$

$$0 \leq V_{tv}^{DAd} \leq V_{tv}^{bidD} : \underline{\mu}_{tv}^{Vd}, \overline{\mu}_{tv}^{Vd}, \ \forall t, v \tag{2o}$$

$$-\underline{F_l} \leq F_{tl} \leq \overline{F}_l : \underline{\vartheta}_{tl}, \overline{\vartheta}_{tl} \quad \forall t, l \tag{2p}$$

$$\sum_n inj_{tn} = 0 \quad , \quad \forall t \tag{2q}$$

$$F_{tl} = \sum_n PTDF_{nl}\, inj_{tn} \quad \forall t, \forall l \quad \}. \tag{2r}$$

The objective function (2a) consists of two terms that represent the negative of MP's profits, as obtained by physical power generation and virtual bid, respectively. Ramp-up and ramp-down constraints of physical generations are represented by (2b) and (2c). Constraints (2d)–(2f) denote the maximum and minimum physical power offers and virtual bids, respectively. Constraint (2g) declares that the virtual bids cannot be simultaneously generation and demand at each time period. Nonnegativity constraints of offers/bids prices are illustrated by (2h). The set of UL decision variables is $\Omega_1^{DA} = \{P_{tib}^{Sbid}, \alpha_{tib}^S, V_{tv}^{bidG}, \alpha_{tv}^{bidG}, V_{tv}^{bidD}, \alpha_{tv}^{bidD}, Ug_{tv}, Ud_{tv}\}$.

The objective of the LL subproblem that represents the DA market clearing model is to minimize the negative of social welfare. The first two terms of the objective function (2i) represent the physical generations offers of the strategic and nonstrategic MPs, respectively. The third term models the physical loads bids and the fourth term denotes the virtual generations and demands bids. Generation-load balance constraint is represented by (2j). Constraints (2k)–(2o) limit the strategic MP's generation power, nonstrategic MP's generation power, loads power, virtual generation, and virtual demand quantities to their corresponding maximum and minimum offers or bids. Power flows of transmission lines, which are calculated by (2r), are bounded by their maximum capacities (4p). The LL decision variable set is stated as $\Omega_2^{DA} = \left\{P_{tib}^S, P_{tjb}^g, P_{tdk}^d, V_{tv}^{DAg}, V_{tv}^{DAd}\right\}$. ISO takes the offer price of physical generation along with the bid prices of virtual transactions as parameters, thus the LL subproblem is linear and convex. Employing the methodology introduced in the subsection IV.A, the single level optimization model of the second stage problem is constructed as expressed in APPENDIX B.

It is worth mentioning that the nonlinear terms in the objective functions (1a) and (2a) are linearized using the SDT approach [35]. Moreover, complementarity nonlinear constraints can be linearized using Big M method. For clarity, these linearized formulations are presented in APPENDIX C.

Note that although the MP's bidding strategy model in FTR auction depends on the DA market outcomes, the actual DA market model is independent of the FTR auction model and no FTR market outcomes is needed to create the bidding strategy model in DA market. To solve this issue, the second term of the objective function (1a) is transferred to the second stage objective function (2a). This way, the required transition information from the first stage problem to the second stage problem will be FTR_e^s, and the objective value of the second stage will be the required data passed from the second stage problem to the first stage problem. This is noticeably depicted in Fig. 7.

C. *Proposed two-stage two-level optimization model*

Having the two-level optimization models for both stages, along with the required transition information between these stages, the final model is formulated as shown:

$$\begin{aligned}\underset{\Delta}{\text{Minimize}} \sum_t \Bigg(& -\sum_f \rho_f FTR_f + \sum_f \sigma_c FTR_c + \sum_f \overline{\tau}_f \overline{FTR}_f \\ & + \sum_c \overline{\tau}_c \overline{FTR}_c + \sum_{tl} \left(\overline{\xi}_l + \underline{\xi}_l\right) \overline{F}_l \\ & - \left(LMP_{t,sink} - LMP_{t,source}\right) FTR_e^s \Bigg) \\ & + \Bigg[\sum_{tib} \Bigg(\lambda_{tib}^S P_{tib}^s + \sum_{t(v\in\psi_n)} \lambda_{tn}^{RT} \left(V_{tv}^{DAg} - V_{tv}^{DAd}\right) \Bigg) \\ & + \Bigg(\sum_{tjb} \lambda_{tjb}^g P_{tjb}^g + \sum_{tjb} \overline{\mu}_{tjb}^G \overline{P}_{tjb}^G - \sum_{tdk} \lambda_{tdk}^d P_{tdk}^d \\ & + \sum_{tdk} \overline{\mu}_{tdk}^D \overline{P}_{tdk}^D + \sum_{tl} \left(\underline{\vartheta}_{tl} + \overline{\vartheta}_{tl}\right) \overline{F}_l \Bigg) \Bigg] \end{aligned} \tag{3a}$$

$$\text{Constraints (C.2), (C.3), (C.5), (C.6)} \tag{3b}$$

where the decision variable set is $\Delta = \{\Omega_1^{FTR}, \Omega_2^{FTR}, \Omega_1^{DA}, \Omega_2^{DA},$ all corresponding dual variables$\}$. Equation (3b) represents the first and second stages' constraints, which are described in APPENDIX C.

V. ILLUSTRATIVE EXAMPLE

To illustrate the mechanism and the functionality of the proposed model, an illustrative example is designed for this section, and it is implemented on 5-bus test system [38].

A. *Data*

To assist the strategic MP in making its joint bidding decisions, other players' data in both FTR auction and DA market are required. Fig. 8 depicts the *sink* and *source* buses of all players' offers/bids. Note that *sink* and *source* buses could be determined based on the DA LMP forecast; however, this paper focuses on the design of the bidding strategy of MPs, and this topic is out of the scope of this research. Strategic MP is assumed to be a buyer in the FTR auction and tends to buy FTR from bus 2 to bus 5 (FTR_5 in Fig. 8). Detailed information of seven players offers/bids in FTR auction is summarized in Table I that represents the FTR number, source and sink buses, players status, bid prices and quantities in different columns.

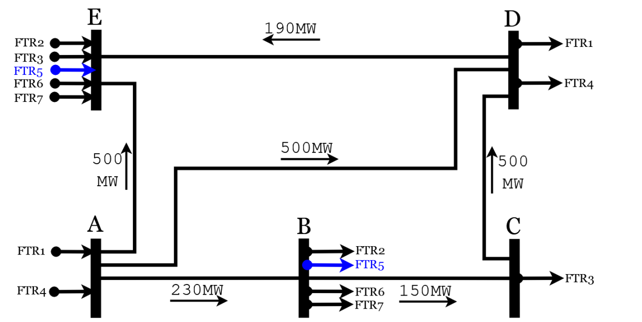

Fig. 8. FTR offers and bids illustration in 5-bus test system

TABLE I. FTR OFFERS/BIDS OF ALL MPS IN THE FTR AUCTION

FTR #	Source (busNo.)	Sink (bus No.)	Status (Buyer/Seller)	Bid Price ($/MWH)	Bid Quantity (MW)
1	1	4	Seller	5	75
2	5	2	Buyer	8	140
3	5	3	Seller	6	120
4	1	4	Buyer	9	110
5	5	2	Buyer	***Variable***	***Variable***
6	5	2	Buyer	8	100
7	5	2	Buyer	10	100

Offers/bids of all physical generators/loads in DA market are depicted in Fig. 9 beside their corresponding elements. Moreover, the transmission lines capacities are displayed on corresponding lines. It is assumed that the strategic MP owns a generator 5 (G5) located at bus "E" with the marginal cost equal to $35/MWH.

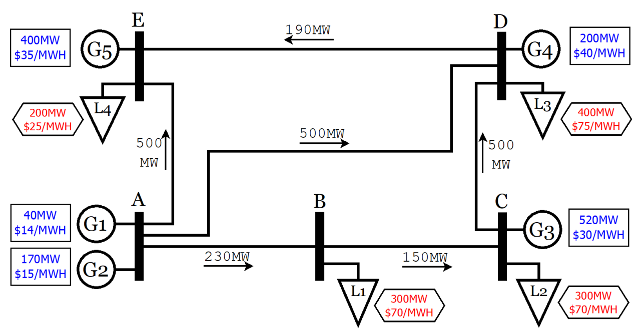

Fig. 9. Offers/bids of all physical generators/loads in DA market

B. *Case Design*

To present the effectiveness of the proposed model, four different cases are designed as follows.

- **Case 1**: the strategic MP bids separately in FTR auction and DA market with the assumption that the accurate prediction of DA LMP difference between *source* and *sink* buses (DLMP) is available. In this case, Model (C) [described in APPENDIX

C] are solved separately to determine the bidding strategies of the MP in FTR auction and DA market.

- **Case 2**: this case is similar to Case 1, except that the accurate DLMP forecast is not available.
- **Case 3**: in Case 1 and Case 2, the strategic MP's bidding decisions in FTR auction are not included in the MP's decision-making process in DA market, which is considered in this case. Therefore, Model (C.1–C.3) is solved at the first step, similar to Case 2, and then the cleared FTR quantity and FTR auction price are passed to the modified version of Model (C.4–C.6) that includes the first stage results in its objective function to capture the bidding decision of MP for the DA market.
- **Case 4**: applies the proposed joint bidding strategy decision making model (Model (3)) that simultaneously optimize the decisions of the MP in FTR auction and DA market.

Note that to emphasize the influence of the virtual bids on the final decisions and profit of the strategic MP, these designed cases are solved twice, with and without considering the virtual bids, and the results are compared afterwards.

C. *Results*

Strategic MP's FTR auction profit, DA market profit, and total profit are illustrated in Fig. 10. Employing inaccurate DA LMP predictions in Case 2 and Case 3 causes negative profits in FTR auction for these cases. However, including the FTR bidding decisions in the second stage of Case 3, makes the MP offer its power with higher price ($53.3/MWH) in the DA market, so the DA LMP at bus 2 will be $70/MWH because of the congestion at line BC. This causes the value of FTR to change from [(55.017 – 44.57) – 30.3 = $(–19.85)/MWH] in Case 2 to [(70 – 53.3) – 30.3 = $(–13.6)/MWH] in Case 3; therefore, MP loses less money from the FTR auction (Table II). Although this action caused lower profit in the DA market because of the MP's lower cleared power (133.18MW) in Case 3 in comparison with that of Case 2 (270.45 MW), the total profit in Case 3 is higher than in Case 2.

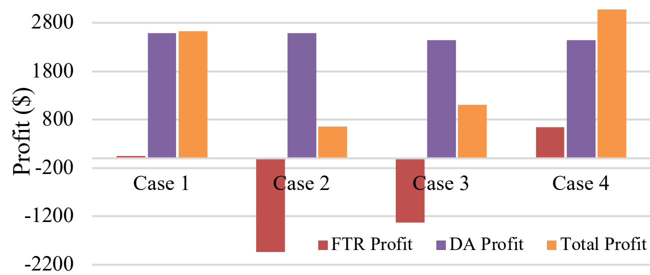

Fig. 10. MP's FTR profit, DA profit, and Total profit in different cases

Comparison between Case 1 and Case 4 declares that although the bidding decisions of MP in FTR auction in both cases are the same (Table II), MP differs its strategy in the DA market (offers its power with higher price) to increase the FTR value from [(55.017 – 44.57) – 10 = $0.45/MWH] in Case 1 to [(70 – 53.3) – 10 = $6.7/MWH] in Case 4, thereby making more profit in the FTR auction. This way, MP loses a small amount of money in the DA market, however, this change in the DA profit is smaller than the MP's FTR auction profit. Put simply, the strategic MP intentionally loses money in the DA market (by its strategic decisions) to increase the FTR value and maximize its total profit.

To present the effect of virtual bids on MP's bidding strategy decision making, it is assumed that the strategic MP is able to submit the maximum virtual bids (generation/load) of 200MW in buses 2 and 5. Real-time prices are predicted to be $30/MWH for all buses. Table III summarizes the results of different cases implementation considering the virtual bids. Comparing the results of Case 2 and Case 3 in this test with the results of the same cases without considering the virtual bids, shows that the MP can make more total profits in both cases. This happens because MP prefers to employ virtual generation at bus 5 instead of submitting the expensive physical generation to alter the DA LMPs.

TABLE II. RESULTS OF DIFFERENT CASES FOR ILLUSTRATIVE EXAMPLE WITHOUT CONSIDERING VIRTUAL BIDS

	Bidding status (Separately /Jointly)	FTR Offer Price ($/MWH)	FTR Cleared Power (MW)	FTR auction Price ($/MWH)	DA Offer Price ($/MWH)	DA Cleared Power (MW)	DA LMP@Bus 5 ($/MWH)	DA LMP@Bus 2 ($/MWH)	FTR Profit ($)	DA Profit ($)	Total Profit ($)
Case 1	Separately	10	94.99	10	44.57	270.45	44.57	55.017	42.46	2588.15	2630.61
Case 2	Separately	30.3	97.37	30.3	44.57	270.45	44.57	55.017	-1934.65	2588.15	653.51
Case 3	Separately		97.37	30.3	53.3	133.18	53.3	70	-1325.4	2436.62	1111.22
Case 4	Jointly	10	94.99	10	53.3	133.18	53.3	70	636.8	2436.62	3073.4

TABLE III. RESULTS OF DIFFERENT CASES FOR ILLUSTRATIVE EXAMPLE WITH CONSIDERING VIRTUAL BIDS

	Bidding status (Separately /Jointly)	FTR Offer Price ($/MWH)	FTR Cleared Power (MW)	FTR auction Price ($/MWH)	DA Offer Price ($/MWH)	DA Cleared Power (MW)	Virtual Gen. Bid Price ($/MWH)	Virtual Gen. Cleared Power (MW)	DA LMP@Bus 5 ($/MWH)	DA LMP@Bus 2 ($/MWH)	FTR Profit ($)	Virtual Bids Profit ($)	DA Profit ($)	Total Profit ($)
Case 1	Separately	10	94.99	10	44.57	69.38	55.017 @ bus 2 44.57 @ bus 5	0.62 @ bus 2 200 @ bus 5	44.57	55.017	42.46	2929.55	663.97	3636
Case 2	Separately	30.3	97.37	30.3	44.57	69.38	55.017 @ bus 2 44.57 @ bus 5	0.62 @ bus 2 200 @ bus 5	44.57	55.017	-1934.65	2929.55	663.97	1658.87
Case 3	Separately		97.37	30.3	53.3	0	70 @ bus 2 0 @ bus 5	77.564 @ bus 2 0 @ bus 5	53.3	70	-1325.4	3102.55	0	1777.15
Case 4	Jointly	10	94.99	10	53.3	0	70 @ bus 2 0 @ bus 5	77.564 @ bus 2 0 @ bus 5	53.3	70	636.8	3102.55	0	3739.3

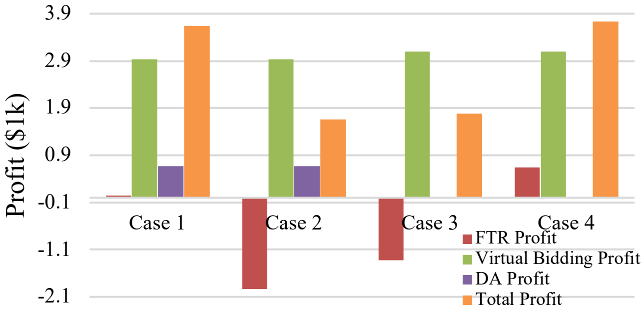

Fig. 11. MP's FTR profit, virtual bidding profit, DA market, and Total profit in different cases

According to Fig. 11, virtual bids provide the ability to make the strategic MP increase the FTR value and also make more profit from virtual bidding in DA market, which results in a higher total profit in Case 4 compared to Case 1. To be more specific, it can be said that the MP prefers to submit the lower virtual generation (77.56MW) at bus 2 with the higher price ($70/MWH) in Case 4 instead of selling 200MW at bus 5 with the lower price ($44.57) in Case 1. This way MP makes more profit from virtual bidding, and at the same time, the FTR value raises to $6.7/MWH.

To further specify the advantages of employing virtual bids, the results of Case 4 without virtual bids (Table II) were compared to the results of Case 4 when the MP employs virtual bids (Table II). Fig. 12 summarizes the comparison between these two results. As shown, virtual bid assists the MP to increase the FTR value with lower cost in the DA market and maximize its total profit. In other words, virtual bidding provides a noticeable opportunity for MP to increase the DA market profit and raise the FTR value more economically.

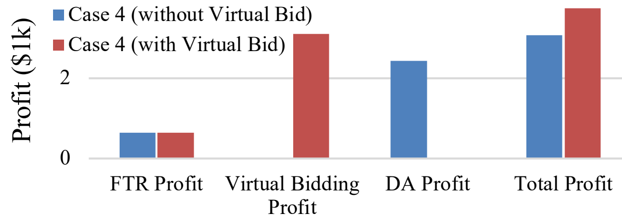

Fig. 12. MP's profit from FTR, virtual bidding, physical generation, and Total profit in Case 4 with and without employing virtual bids

VI. CASE STUDY

To demonstrate the effectiveness of the proposed joint bidding strategy model and the value of utilizing the virtual bidding, Model (3) is solved for the strategic generating company participating in the FTR auction and DA market in the 24-bus test system [39].

A. Data

Many ISOs in the US publish some of the historical market data such the market clearing results, historical bids and offers, and etc. for different electricity markets [40 - 44]. Such information may be used by the strategic MP to formulate, estimate and forecast the parameters needed for the decision making of the MP. In this work, all data are selected based on generator characteristics [45] and financial constraints [46] to be aligned with real-world data. It is assumed that 16 sellers and 24 buyers submit their offers and bids into the FTR auction, as exemplified in Table IV. The strategic MP plans to purchase eight FTRs from different *sink* and *source* buses in this auction.

Furthermore, the strategic MP is assumed to own 7 generating units in different locations of the system; their maximum capacities and marginal costs are listed in Table V.

TABLE IV. FTR OFFERS/BIDS OF ALL MPS IN THE FTR AUCTION FOR 24-BUS SYSTEM

FTR #	Source (bus #)	Sink (bus #)	Status	Bid Price ($/MWH)	Bid Quantity (MW)	FTR #	Source (bus #)	Sink (bus #)	Status	Bid Price ($/MWH)	Bid Quantity (MW)
1	23	12	Seller	3	50	21	12	9	Seller	8	120
2	23	13	Seller	3	80	22	11	10	Seller	7	140
3	23	12	Buyer	***Variable***	***Variable***	23	24	3	Seller	6	120
4	23	12	Buyer	15	100	24	24	3	Buyer	***Variable***	***Variable***
5	23	12	Buyer	10	120	25	24	3	Buyer	13	50
6	23	13	Buyer	15	110	26	24	3	Buyer	14	75
7	23	13	Buyer	***Variable***	***Variable***	27	14	11	Seller	4	190
8	23	13	Buyer	9	90	28	15	16	Seller	3	120
9	16	14	Buyer	***Variable***	***Variable***	29	7	8	Buyer	***Variable***	***Variable***
10	16	14	Buyer	11	200	30	7	8	Seller	7.5	120
11	15	21	Buyer	***Variable***	***Variable***	31	7	8	Buyer	15	60
12	16	14	Buyer	12	150	32	24	15	Seller	4	240
13	16	14	Seller	4	100	33	22	21	Buyer	***Variable***	***Variable***
14	15	21	Buyer	11.5	300	34	12	10	Buyer	12	200
15	15	21	Buyer	10	120	35	12	10	Seller	6.5	80
16	15	21	Seller	3.5	50	36	16	19	Seller	8	100
17	17	16	Seller	4	60	37	17	18	Buyer	10.5	200
18	22	17	Buyer	***Variable***	***Variable***	38	17	22	Seller	5.5	150
19	17	16	Buyer	9.5	160	39	11	10	Buyer	15	50
20	11	9	Seller	3.5	105	40	24	15	Buyer	12.5	75

TABLE V. STRATEGIC GENERATING UNITS DATA

Gen #	$\overline{P}^S_{tib}$ (MW)	λ^S_{tib} ($/MWH)	(Bus #)
G1	76	15	1
G2	76	15	2
G3	400	7	7
G4	70	4	13
G5	197	20	16
G6	155	13	21
G7	155	13	23

Predicted offer quantities and prices for 25 other generators in DA market are summarized in Table VI. These are presumed to be the same for all periods of time. Locations and maximum bid quantities of 17 loads in this system are shown in Table VII, and the bid prices in different time periods are depicted in Fig. 13. Note that three different bid price profiles used for different loads in different location.

TABLE VI. GENERATION UNITS' OFFER QUANTITIES AND PRICES

Gen #	$\overline{P}^G_{tjb}$ (MW)	λ^G_{tjb} (\$/MWH)	(Bus #)	Gen #	$\overline{P}^G_{tjb}$ (MW)	λ^G_{tjb} (\$/MWH)	(Bus #)
G1	21	16	1	**G14**	12	27	15
G2	21	16	1	**G15**	155	13	15
G3	21	16	1	**G16**	76	15	15
G4	21	16	2	**G17**	70	4	18
G5	90	18	2	**G18**	70	4	22
G6	90	22	2	**G19**	70	4	22
G7	90	22	7	**G20**	70	4	22
G8	155	13	7	**G21**	70	4	22
G9	155	13	13	**G22**	90	22	22
G10	12	27	13	**G23**	155	13	22
G11	12	27	15	**G24**	90	22	23
G12	12	27	15	**G25**	197	20	23
G13	12	27	15				

TABLE VII. LOAD BID QUANTITIES AND PRICES

Load #	$\overline{P}^D_{tdk}$ (MW)	(Bus #)	Load #	$\overline{P}^D_{tdk}$ (MW)	(Bus #)
L1	105	1	**L10**	188	10
L2	92	2	**L11**	255	13
L3	172	3	**L12**	188	14
L4	73	4	**L13**	305	15
L5	71	5	**L14**	98	16
L6	133	6	**L15**	323	18
L7	119	7	**L16**	174	19
L8	165	8	**L17**	126	20
L9	167	9			

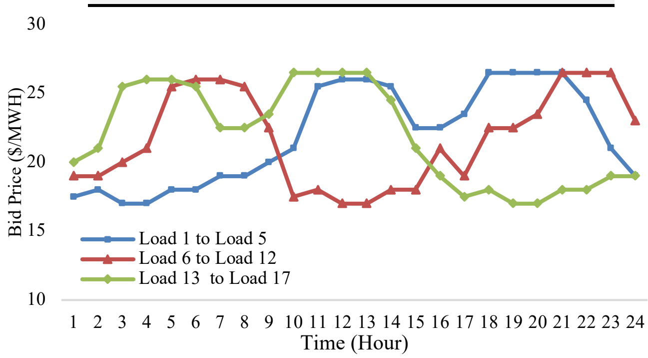

Fig. 13. Load bid price profile

B. Results

The offering strategy problem is solved for the four designed cases introduced in the previous section, and the models are tested for the strategic MP with and without virtual bidding capability. Fig. 14 shows the GenCo's profits in FTR auction, DA market and its total profit when the virtual bidding capability is not considered. Inaccurate DA LMPs forecast causes lower FTR profits in Case 2 and Case 3, which results in the lower total profit for these cases. The DA profit of MP is lower in Case 4 compared to Case 1; however, this is opposite for the FTR profit regarding these cases. This declares that the MP strategically loses a small amount of money in the DA market to increase the FTR value and optimize its total profit.

Applying the virtual bidding, MP finds more opportunities to raise the FTR value and maximize its total profit. More specifically, Fig. 15 presents the MP's profits from both physical generation and virtual bidding in DA market are higher in Case 1 than in Case 4. However, with this strategy, MP increases its FTR profit from \$85k to approximately \$265k, and as a result, makes more total profit. Note that it is assumed that the MP submits its virtual transactions from various locations (buses 3, 7, 11, 14, 17, and 22 in this study), and the forecasted real-time LMPs are assumed to be \$20/MWH for all time periods.

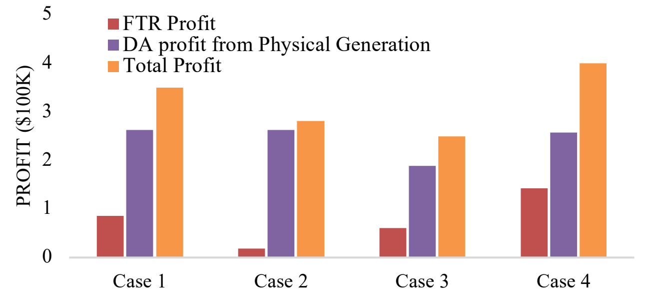

Fig. 14. MP's profits in FTR auction, DA market as well as its total profit without considering the virtual bids

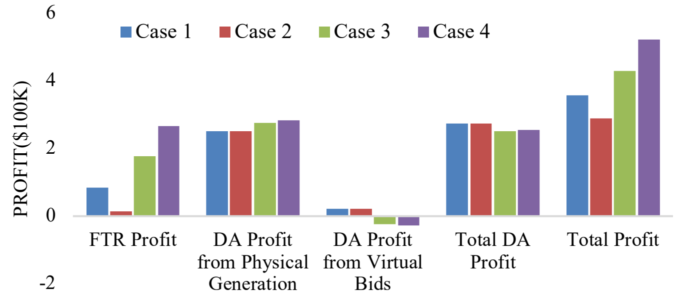

Fig. 15. MP's profits from FTR, virtual bidding, physical generation along with its total profit

Comparing Case 4 from two tests (with and without virtual bidding) in Fig. 16 depicts that the presence of virtual bids can help the MP manipulate the FTR value and increase the FTR profit and improve the DA market profit.

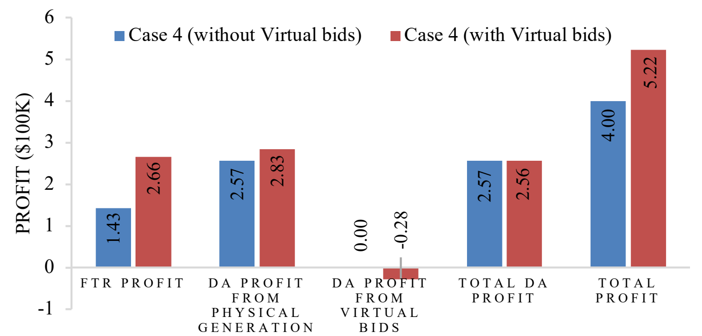

Fig. 16. MP's profits from FTR, virtual bidding, and physical generation as well as its total profit in Case 4 with two tests (with and without considering virtual bids)

VII. CONCLUSION

This paper proposed a joint offering strategy model for a strategic GenCo with virtual bidding capability to participate in both FTR auction and DA market, in markets where simultaneous participation in virtual transactions and FTR auction is allowed. First, using a simple example, it was demonstrated that virtual bids can be used to manipulate the FTR values. Next, the proposed model was developed and solved by employing the SDT and KKT optimality conditions. Further, various cases were designed to illustrate the effectiveness of the model and value of utilizing virtual bidding in the MP's offering strategy decision making process. In summary, the conclusions of this work are listed below.

- A strategic GenCo faces complicated decision making due to the competing goals. It must delicately balance multiple objectives in the decision-making, such as (a) the tradeoff between the profits in FTR auction and the DA market due to the impact of DA LMP on FTR revenue; (b) tradeoff between profits of virtual bids and physical generation in the DA market due to both affecting DA LMPs; and (c) tradeoff between the FTR quantity and its impact on FTR auction price as both relate to FTR profit.

- Based on the proposed model, the MP may choose a tactic that, by using virtual bids and its physical generation, can make higher profits through joint participation in FTR and DA markets. In fact, virtual bidding can create an opportunity for MP to increase the FTR value by manipulating the DA LMP at specific locations of the system. In some cases, manipulated DA LMP can result in reduced or negative profit from virtual bidding. Despite the resulting profit in DA market is reduced, the profit from FTR market is much increased and so is the total profit.

A strategic MP faces multiple uncertainties such as forecasted RT prices and other players' offers/bids in the decision-making process. Considering these uncertain parameters in the proposed model and applying an appropriate solution methodology for this problem can be the focus of future work.

APPENDIX A

Mathematical Problem with Equilibrium Constraints (MPEC) model for the first stage problem.

Objective Function (1a) (A.1)

s.t:

Constraints (1b), (1c), (1i) and (1j) (A.2)

$$-\rho_e^s + \overline{\tau}_e^S - \underline{\tau}_e^S + MCP_e = 0, \quad \forall e \tag{A.3}$$

$$-\rho_f + \overline{\tau}_f - \underline{\tau}_f + MCP_f = 0, \forall f \in \{N_{pur} - e\} \tag{A.4}$$

$$\sigma_c + \overline{\tau}_c - \underline{\tau}_c - MCP_c = 0, \quad \forall c \in N_{sell} \tag{A.5}$$

$$0 \le FTR_e^{Sbid} - FTR_e^s \perp \overline{\tau}_e^S \ge 0, \quad \forall e \tag{A.6}$$

$$0 \le FTR_e^s \perp \underline{\tau}_e^S \ge 0, \quad \forall i \tag{A.7}$$

$$0 \le \overline{FTR}_f - FTR_f \perp \overline{\tau}_f \ge 0, \quad \forall f \tag{A.8}$$

$$0 \le FTR_f \perp \underline{\tau}_f \ge 0, \quad \forall f \tag{A.9}$$

$$0 \le \overline{FTR}_c - FTR_c \perp \overline{\tau}_c \ge 0, \ \forall c \in N_{sell} \tag{A.10}$$

$$0 \le FTR_c \perp \underline{\tau}_c \ge 0, \quad \forall c \in N_{sell} \tag{A.11}$$

$$0 \le \overline{F}_l - LF_l^{ex} - LF_l \perp \overline{\xi}_l \ge 0, \quad \forall l \tag{A.12}$$

$$0 \le \overline{F}_l + LF_l^{ex} + LF_l \perp \underline{\xi}_l \ge 0, \quad \forall l \tag{A.13}$$

In (A), the nonlinear objective function is the same as in Model (1). Constraint (A.2) replicates (1b), (1c), (1i) and (1j) constraints. The first derivatives of the Lagrangian function with respect to the decision variables are shown in (A.3)–(A.5), and the nonlinear complementarity constraints that result from the inequality constraints of the LL subproblem of Model (1) are shown in (A.6)–(A.13).

APPENDIX B

Mathematical Problem with Equilibrium Constraints (MPEC) model for the second stage problem.

Objective Function (2a) (B.1)

s.t:

Constraints (2b)–(2h) (B.2)

$$\alpha_{tib}^s - LMP_{tn} + \overline{\mu}_{tib}^S - \underline{\mu}_{tib}^S = 0, \forall t, i \in \psi_n, b \tag{B.3}$$

$$\lambda_{tjb}^g - LMP_{tn} + \overline{\mu}_{tjb}^G - \underline{\mu}_{tjb}^G = 0, \forall t, j \in \psi_n, b \tag{B.4}$$

$$-\lambda_{tdk}^d + LMP_{tn} + \overline{\mu}_{tdk}^D - \underline{\mu}_{tdk}^D = 0, \quad \forall t, d \in \psi_n, k \tag{B.5}$$

$$\alpha_{tv}^{bidG} - LMP_{tn} + \overline{\mu}_{tv}^{Vg} - \underline{\mu}_{tv}^{Vg} = 0, \forall t, v \in \psi_n \tag{B.6}$$

$$-\alpha_{tv}^{bidD} + LMP_{tn} + \overline{\mu}_{tv}^{Vd} - \underline{\mu}_{tv}^{Vd} = 0, \quad \forall t, v \in \psi_n \tag{B.7}$$

Constraints (2j) and (2q) and (2r) (B.8)

$$0 \le P_{tib}^S \perp \underline{\mu}_{tib}^S \ge 0, \quad \forall t, i, b \tag{B.9}$$

$$0 \le P_{tib}^{Sbid} - P_{tib}^S \perp \overline{\mu}_{tib}^S \ge 0, \ \forall t, i, b \tag{B.10}$$

$$0 \le P_{tjb}^g \perp \underline{\mu}_{tjb}^G \ge 0, \quad \forall t, j, b \tag{B.11}$$

$$0 \le \overline{P}_{tjb}^G - P_{tjb}^g \perp \overline{\mu}_{tjb}^G \ge 0, \ \forall t, j, b \tag{B.12}$$

$$0 \le P_{tdk}^d \perp \underline{\mu}_{tdk}^D \ge 0, \quad \forall t, d, k \tag{B.13}$$

$$0 \le \overline{P}_{tdk}^D - P_{tdk}^d \perp \overline{\mu}_{tdk}^D \ge 0, \ \forall t, d, k \tag{B.14}$$

$$0 \le V_{tv}^{DAg} \perp \underline{\mu}_{tv}^{Vg} \ge 0, \quad \forall t, v \tag{B.15}$$

$$0 \le V_{tv}^{DAd} \perp \underline{\mu}_{tv}^{Vd} \ge 0, \quad \forall t, v \tag{B.16}$$

$$0 \le V_{tv}^{bidG} - V_{tv}^{DAg} \perp \overline{\mu}_{tv}^{Vg} \ge 0, \quad \forall t, v \tag{B.17}$$

$$0 \le V_{tv}^{bidD} - V_{tv}^{DAd} \perp \overline{\mu}_{tv}^{Vd} \ge 0, \quad \forall t, v \tag{B.18}$$

$$0 \le F_{tl} + \overline{F}_l \perp \underline{\vartheta}_{tl} \ge 0 \quad \forall t, l \tag{B.19}$$

$$0 \le \overline{F}_l - F_{tl} \perp \overline{\vartheta}_{tl} \ge 0 \quad \forall t, l \tag{B.20}$$

In (B), the objective function and the UL constraints (2b)–(2h) are duplicated in (B.1) and (B.2). The first derivative of Lagrangian function with respect to the decision variables are denoted by (B.3)–(B.7). The equality constraints (2j), (2q), and (2r) are summarized in (B.8). Constraints (B.9)–(B.20) represent the nonlinear complementarity constraints regarding the inequality constraints (2k)–(2p).

APPENDIX C

To linearize the first nonlinear term of the objective function (1a (or A.1)), the SDT approach is employed, which is well described in [35]. Thus, applying these methods to (A) results in the following model with linear constraints.

$$\boldsymbol{OF_1} = \underset{\Omega_1^{FTR}, \Omega_2^{FTR}}{\text{Minimize}} \sum_t \Bigg(-\sum_f \rho_f FTR_f + \sum_f \sigma_c FTR_c + \sum_f \overline{\tau}_f \overline{FTR}_f + \sum_c \overline{\tau}_c \overline{FTR}_c + \sum_{tl} \left(\overline{\xi}_l + \underline{\xi}_l\right) \overline{F}_l - \left(LMP_{t,sink} - LMP_{t,source}\right) FTR_e^s \Bigg) \tag{C.1}$$

s.t:

Constraints (A.2)–(A.5) (C.2)

Linearized form of (A.6)–(A.13) (C.3)

Moreover, complementarity nonlinear constraints can be linearized using Big M method. Thus, each of the equations of $0 \le X_{ti} \perp d_{ti}(x) \ge 0$ can be rewritten as follows.

$$0 \le X_{ti} \le M_{ti}\, \omega_{ti}, \ \ 0 \le d_{ti}(x) \le (1 - \omega_{ti})\, M_{ti}$$

where M_{ti} is a large number and ω_{ti} is a binary variable.

Applying SDT and Big M methods, the equivalent linear formulation of the problem (B) is obtained as follows.

$$\boldsymbol{OF_2} = \underset{\Omega_1^{DA}, \Omega_2^{DA}}{\text{Minimize}} \left[\sum_{tib} \left(\lambda_{tib}^{S} P_{tib}^{s} + \sum_{t(v \in \psi_n)} \lambda_{tn}^{RT} \left(V_{tv}^{DAg} - V_{tv}^{DAd} \right) \right) + \left(\sum_{tjb} \lambda_{tjb}^{g} P_{tjb}^{g} + \sum_{tjb} \overline{\mu}_{tjb}^{G} \overline{P}_{tjb}^{G} - \sum_{tdk} \lambda_{tdk}^{d} P_{tdk}^{d} + \sum_{tdk} \overline{\mu}_{tdk}^{D} \overline{P}_{tdk}^{D} + \sum_{tl} \left(\underline{\vartheta}_{tl} + \overline{\vartheta}_{tl} \right) \overline{F}_{l} \right) \right] \quad (C.4)$$

s.t:

$$\text{Constraints (B.2)–(B.8)} \quad (C.5)$$

$$\text{Linearized form of (B.9)–(B.20)} \quad (C.6)$$

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