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Developing Robust Bidding Strategy for Virtual Bidders in Day-Ahead Electricity Markets

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ABSTRACT Purely financial players without any physical assets can participate in day-ahead electricity markets as virtual bidders. They can arbitrage the price difference between day-ahead (DA) and real-time (RT) markets to maximize profits. Virtual bidders encounter various monetary risks and uncertainties in their decision-making due to the high volatility of the price difference. Therefore, this paper proposes a max-min two-level optimization model to derive the optimal bidding strategy of virtual bidders. In this model, the risks of uncertainties associated with the rivals' strategies and RT market prices are managed by robust optimization. The proposed max-min two-level model is turned into a single-level mixed integer linear programming model through duality theory (DT), strong duality theory (SDT), and Karush-Kuhn-Tucker (KKT) conditions. An illustrative case is designed to demonstrate the advantages of the proposed model over the deterministic model. Moreover, case studies on the IEEE 24-bus test system validate the applicability of the proposed model.

INDEX TERMS Bidding strategy, duality theory, robust optimization, uncertainty, virtual bidding.

NOMENCLATURE

A. SETS AND INDICES

t	Index for time periods.
i	Index for virtual participants.
j	Index for generating units.
b	Index for generation blocks.
d	Index for demands.
k	Index for demand blocks.
l	Index for transmission lines.
Ω, Ξ	Decision variable sets for the upper/lower level subproblems, respectively.
Γ	Uncertain variables set.
Φ	Dual variables set for the lower-level subproblem.

B. PARAMETERS

λ_{tn}^{RT}	Predicted real-time price at time t at bus n.
$\bar{P}_{tjb}^{G}$	Predicted capacity of unit j of the other generating units at time t.
$\bar{P}_{tdk}^{D}$	Predicted demand d at time t.
λ_{tjb}^{G}	Predicted marginal cost of unit j of the other generating units at time t.
λ_{tdk}^{D}	Predicted marginal utility of demand d at time t.
ζ_{n}^{RT}	Robustness parameter of RT price at bus n.
σ_{jb}^{G}, σ_{dk}^{D}	Robustness parameter of offered/bid quantities of block b of other MPs j and block k of demand d.
τ_{jb}^{G}, τ_{dk}^{D}	Robustness parameter of offered/bid prices of block b of other MPs j and block k of demand d.
V_{ti}^{budget}	maximum amount of generation/consumption of virtual participant i at time t.
$\bar{C}_{l}$	Transmission capacity of line l.
H_{nl}	Power transfer distribution factor (PTDF).

C. VARIABLES

α_{ti}^{bidG}, α_{ti}^{bidD}	Bid Price of virtual participant generation/ consumption at time t.
V_{ti}^{bidG}, V_{ti}^{bidD}	Bid generation/consumption quantity of virtual participant i at time t.

V_{ti}^{DAg}, V_{ti}^{DAd}	Cleared generation/consumption quantity of virtual participant i at time t.
P_{tjb}^{G}	Cleared power produced by unit j of the other generating units at time t.
P_{tdk}^{D}	Cleared power consumed by demand d at time t.
$\Delta\lambda_{tn}^{RT}$	Deviation in prediction of real-time LMP.
ΔP_{tjb}^{G}, ΔP_{tdk}^{D}	Variation of the forecasted offered/bid generation/demand quantities.
$\Delta\lambda_{tjb}^{G}$, $\Delta\lambda_{tdk}^{D}$	Variation of the forecasted offered/bid generation/demand prices.
Ug_{ti}, Ud_{ti}	Binary Variables represent the virtual generation or load.

D. DUAL VARIABLES

λ_{tn}^{DA}	Generation-demand equilibrium at time t at bus n (in DA market).
$\underline{\mu}_{ti}^{V}$	Minimum bid quantity of virtual bidder i at time t.
$\bar{\mu}_{ti}^{V}$	Maximum bid quantity of virtual bidder i at time t.
$\underline{\mu}_{tjb}^{G}$	Minimum generation of block b of unit j at time t.
$\bar{\mu}_{tjb}^{G}$	Capacity of unit j at time t.
$\underline{\mu}_{tdk}^{D}$	Minimum load power of demand d at time t.
$\bar{\mu}_{tdk}^{D}$	Maximum load power of demand d at time t.
$\underline{\vartheta}_{tl}$	Line l capacity at time t and negative way.
$\bar{\vartheta}_{tl}$	Line l capacity at time t and positive way.
ρ, η, θ, χ	Lagrangian coefficients of the lower-level optimization constraints.

I. INTRODUCTION

VIRTUAL traders, or virtual bidders, are purely financial participants in the electricity market, who can submit their bids/offers into the day-ahead (DA) market without the compulsion to consume/produce the actual power in the real-time (RT) market. In recent years, these transactions which are designed as decrement bids (DECs) or increment offers (INCs), have been considered as part of the electricity market design [1].

Virtual bids contributed around 6% of all transactions in the Midwest Independent Transmission System Operator (MISO) in 2010 and 2011 [2], and are generally employed to decrease the gap between the DA and RT markets' prices and increase the liquidity of the markets. The values of various flexible resources, including virtual bids, were evaluated in [3] through four different two-settlement market-clearing models, and it is confirmed that the errors of deterministic DA scheduling can be amended by virtual bids. The advantages and disadvantages of virtual bids in electricity market environments have been summarized in [4], which stated that besides the benefits of virtual bids, they may increase the risks for market manipulation in some parts of the system. A model reported in [5] presents the motivations for market participants (MPs) to place virtual bids at the specific buses which are tied to the financial transmission right (FTR) position owned by the MPs. It is shown that deliberate non-optimal virtual bids to manipulate market prices increase the price difference between DA and RT markets, and thus, create market inefficiencies. To analyze the MP's cross-product manipulation in three sequential markets, a three-stage stochastic game theoretic model has been presented in [6], which applied the numerical simulation to evaluate the influence of this manipulation on price convergence in a two-bus system. An algorithmic trading strategy for virtual bids in power markets using a data-driven approach has been presented in [7]. To optimize the submitting virtual bids, a mixture density network (MDN) model was utilized to predict the price differences between the DA and RT markets. Authors in [8] employed virtual bidding to accommodate the PV solar power producers to optimize their bidding strategies in the electricity market environment, and it is declared that virtual bidding may provide additional risk for risk-taker PV producers. Although multiple works have been done on virtual bids to present the strengths and weaknesses of utilizing these transactions, very limited works has been carried out to study the bidding strategy of this newer type of market participants [9].

In the restructured electricity system, strategic bidding helps MPs to improve their behaviors and maximize their payoffs. The existing approaches for bidding strategy design differ for the price-taker and price-maker MPs. Price-taker MP, whose action does not alter the market outcomes, models the market with the DA market price prediction to optimize its bidding behavior [10], [11]. However, price-maker MP requires a more complicated methodology to design its strategy since its actions impact the market outcomes, thus, it needs to model the rival MPs as well as the market clearing process. [12] applied a MILP model to address the offering strategy problem for a price-maker generator who participates in a DA electricity market in which two distinct methods, based on the nodal and shift factor formulation, were used to model the transmission system. The binary expansion method is employed in [13] to solve the bi-level bidding strategy problem with stepwise offers for the price-maker generator.

In almost all methods existent for the bidding strategy design, there are various uncertainty sources, such as market prices, demand, rivals' strategies, and renewable energy generations, that impact the bidding strategy. The uncertain behavior of rival generators and consumers were modeled probabilistically in [14] and, to find the optimal offering strategy of the price-maker MP, Monte Carlo simulation was employed to compute the expected profit. To derive the optimal bidding strategy for a strategic generation company, a stochastic bi-level optimization problem has been modeled in [15], which modeled the uncertainties of consumers' bids and rival generators' offers through multiple scenarios. In [16], a self-scheduling model is employed to design the bidding strategy of price-maker energy storage and evaluate the potential arbitrage benefits of these resources in the Alberta electricity market using the historical hourly demand and generation price quota curves (DPGCs and GPDCs). To capture the optimal offering decisions of a strategic wind power producer in the DA and balancing markets, a two-stage stochastic model is presented in [17], in which scenario-based modeling is applied to model the uncertainties associated with the wind productions, other players' behaviors, and market price. Optimal bid prices and quantities of a generating company are derived in [18] using a self-organizing hierarchical particle swarm optimization, in which a risk index based on mean-standard deviation ratio (MSR) is optimized, and Monte Carlo simulation is applied to mimic the other MPs' behaviors in the electricity market.

Among all current methods for addressing the uncertainties, robust optimization (RO), which is independent of the probability distribution function (PDF) of the parameters and assumes uncertain intervals around the predicted parameters, has become an appropriate choice for studies with high level of uncertainties and insufficient data for an accurate forecast of PDFs. Recently, the RO method has been widely used for the bidding strategy design problems. A two-stage RO method is utilized in [19] to design the offering strategy of a price-taker virtual power plant (VPP) consisting of a wind power producer, energy storage, and a number of demands taking part in the DA and RT markets. To obtain the optimal strategies of wind power producer and wind-storage aggregator, who are assumed to act as price-makers in the DA market and as deviators in the balancing market, [20, 21] presented a multi-stage distributionally RO (DRO) model and its value was confirmed by the different case studies performed on the modified Swiss system and Nordpool. To address the uncertainties associated with wind power generation and loads, and to obtain the optimal behavior of a virtual power plant, a stochastic adaptive RO method has been presented in [22]. The optimal bidding strategy of a hybrid power plant, which acts as a price-maker in the DA market and a price-taker in the balancing market, has been derived in [23]. It addressed the uncertainty of the price quota curve (PQC) with the RO. [24] utilized the bi-level RO model to optimally design a plug-in electric vehicle (PEV) charging station, in which the lower-level problem modeled the strategic behavior of PEV owners. The optimal bidding curve of a price-maker energy storage facility in the DA market was captured in [25], which employed the robustly modeled generation and demand price quota curves to consider the effect of the energy storage power on market prices. The optimal behavior of a price-maker microgrid aggregator (MGA) using a RO model to address the uncertainties related to renewable generation was presented in [26] and showed that the presented model can improve the MGA profits.

In this paper, we proposed the max-min two-level optimization model for the purely financial player who plays either generation or load in the DA market. The virtual bidder's payoff is maximized in the upper-level subproblem, and the market-clearing procedure is modeled in a lower-level subproblem. In order to avoid the difficulties facing stochastic optimization, such as the expensive computation and the dependency on accurate predictions of the PDFs of uncertain parameters [27], the RO approach is employed in this paper to model the uncertainties of other MPs' offers/bids and RT market prices. Also, the RO approach can guarantee feasibility in all scenarios, while stochastic optimization cannot. The proposed model is turned into its equivalent linear single-level problem employing the KKT conditions, duality theory, and strong duality theory (SDT). Thus, the main contributions of this paper are twofold:

1) It is the first work to employ robust optimization to develop a day-ahead market bidding strategy for purely financial virtual bidders. The proposed model allows the virtual bidder to maximize its profit by considering its flexibility of being either generation or load at different locations of the system. Robust optimization is used to handle the uncertainties associated with other players' offer/bids (quantity and price) and RT market LMPs.

2) Based on the KKT conditions, SDT and big-M method, the proposed max-min two-level model is equivalently linearized and transformed into a mixed-integer linear program (MILP), making it solvable by accessible commercial solvers.

The rest of this paper is organized as follows: Section II introduces virtual bidding and its impact on DA market prices. Section III presents the proposed robust two-level and its corresponding MPEC models. Section IV provides an illustrative example. Case study results are discussed in Section V, and Section VI presents the conclusion.

II. ROBUST BIDDING STRATEGY FOR VIRTUAL BIDDER

A. VIRTUAL BIDDING

Virtual bidders, also known as virtual arbitragers, may have no physical assets and can participate in energy transactions in the DA market. If a virtual bidder is cleared by the ISO to buy (or sell) energy in the DA market for certain time periods, in the ISO two-stage settlement it will be automatically considered to sell (or buy) the same amount of energy in the RT market for the same time periods. As discussed in [9], virtual bidders can improve the market's ability to manage the forecast errors by increasing the liquidity of the market.

Assuming that a virtual bidder predicts the RT price to be higher than the predicted DA price, there will be an opportunity for the virtual bidder to arbitrage between the DA and RT markets by buying a certain amount of energy in the DA market at a DA market price and selling the same amount of energy in the RT market at a RT market price. As a result of this virtual bidder participation, the DA market price may increase due to the increased load cleared in the DA market. Consequently, the difference between DA and RT prices may become smaller, as illustrated in Fig. 1. Therefore, virtual bidder participation in energy markets may reduce the price gap between DA and RT markets, which is considered an improvement in market convergence.

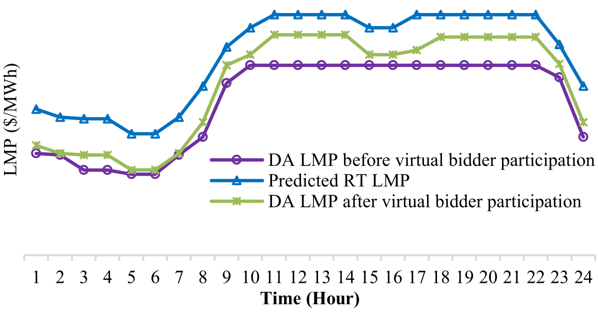

FIGURE 1. The effect of virtual transactions on DA price and DA/RT price difference.

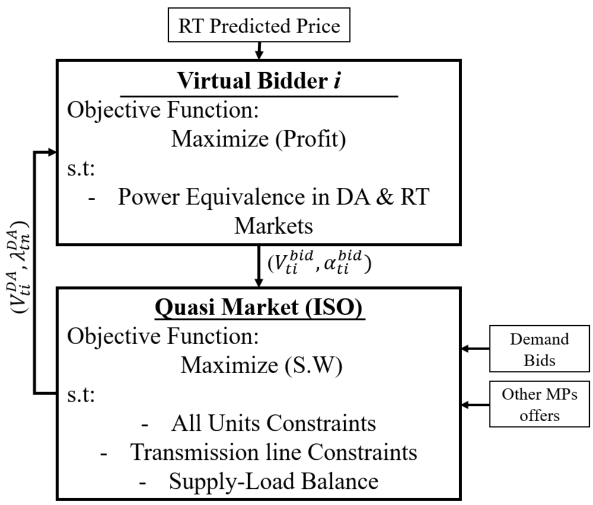

FIGURE 2. Proposed two-level model.

B. MODEL STRUCTURE

The upper-level subproblem of the proposed two-level approach represents the profit maximization of a virtual bidder, whose decisions (virtual bid quantity and price $(V^{bid}_{ti}, \alpha^{bid}_{ti})$) are then passed to the lower-level subproblem. The lower-level subproblem represents the quasi market where energy and market prices are cleared on an hourly granularity on a daily basis. The market results (i.e., cleared virtual quantity and market price $(V^{DA}_{ti}, \lambda^{DA}_{tn})$) are fed back to the upper-level subproblem, which provides a closed loop response of the virtual bidder decision on market price (Fig. 2).

To optimize its decision, the virtual bidder needs to consider various parameters, including the quantities and prices of other generators'/loads' offers/bids, as well as RT market prices. All these parameters play a critical role in the virtual bidder's ultimate payoff. For instance, the DA market price, which is utilized by virtual bidder to estimate its DA profit, may alter as a result of various rivals' offers/bids. Furthermore, the RT price assists the virtual bidder in evaluating the DA profit vs. the RT profit and deciding whether to be virtual generation or virtual demand in the DA market. As these parameters are unknown to the virtual bidder, they need to be forecasted or estimated. However, making a precise prediction is practically impossible. Therefore, to consider the risks of the forecasted uncertainty sources, a robust optimization approach is employed in this paper, which is widely applied by the risk-averse market participants [28]. In this approach, a confidence interval needs to be introduced around an uncertain parameter, then the worst-case scenario of uncertain circumstances within this permissible limit is assessed [25]. Therefore, the proposed max-min two-level optimization model can be formulated as follows.

C. PROPOSED ROBUST OPTIMIZATION MODEL

1) UPPER-LEVEL

Maximize the Profit (Model (1))

$$\min_{\Omega}\max_{\Gamma} \sum_{t} \sum_{(i\in\psi_n)} \left[\lambda^{DA}_{tn} - \left(\lambda^{RT}_{tn} + \Delta\lambda^{RT}_{tn}\right)\right](V^{DAg}_{ti} - V^{DAd}_{ti}) \tag{1a}$$

Subject to:

$$0 \le V^{bidG}_{ti} \le V^{budget}_{ti} Ug_{ti}, \quad \forall t,\ \forall i \tag{1b}$$

$$0 \le V^{bidD}_{ti} \le V^{budget}_{ti} Ud_{ti}, \quad \forall t,\ \forall i \tag{1c}$$

$$Ug_{ti} + Ud_{ti} \le 1, \quad \forall t, \forall i \tag{1d}$$

$$\alpha^{bidG}_{ti} \ge 0, \alpha^{bidD}_{ti} \ge 0, \quad \forall t,\ \forall i \tag{1e}$$

$$-\zeta^{RT}_{n}\lambda^{RT}_{tn} \le \Delta\lambda^{RT}_{tn} \le \zeta^{RT}_{n}\lambda^{RT}_{tn} \tag{1f}$$

$$-\sigma^{G}_{jb}\bar{P}^{G}_{tjb} \le \Delta P^{G}_{tjb} \le \sigma^{G}_{jb}\bar{P}^{G}_{tjb} \tag{1g}$$

$$-\sigma^{D}_{dk}\bar{P}^{D}_{tdk} \le \Delta P^{D}_{tdk} \le \sigma^{D}_{dk}\bar{P}^{D}_{tdk} \tag{1h}$$

$$-\tau^{G}_{jb}\lambda^{G}_{tjb} \le \Delta\lambda^{G}_{tjb} \le \tau^{G}_{jb}\lambda^{G}_{tjb} \tag{1i}$$

$$-\tau^{D}_{dk}\lambda^{D}_{tdk} \le \Delta\lambda^{D}_{tdk} \le \tau^{D}_{dk}\lambda^{D}_{tdk} \tag{1j}$$

2) LOWER-LEVEL

Quasi Day Ahead Market (Model (2))

$$\begin{aligned}\min_{\Xi} \sum_{t} \Bigg(& \sum_{i} (\alpha^{bidG}_{ti} V^{DAg}_{ti} - \alpha^{bidD}_{ti} V^{DAd}_{ti}) \\ & + \sum_{j}\sum_{b} (\lambda^{G}_{tjb} + \Delta\lambda^{G}_{tjb}) P^{G}_{tjb} \\ & - \sum_{d}\sum_{k} (\lambda^{D}_{tdk} + \Delta\lambda^{D}_{tdk}) P^{D}_{tdk} \Bigg) \\ & \Xi = \{V^{DAg}_{ti}, V^{DAd}_{ti}, P^{G}_{tjb}, P^{D}_{tdk}\}\end{aligned} \tag{2a}$$

Subject to:

$$\sum_{i} (V^{DAg}_{ti} - V^{DAd}_{ti}) + \sum_{j}\sum_{b} P^{G}_{tjb} = \sum_{d}\sum_{k} P^{D}_{tdk}, : \lambda^{DA}_{tf}, \quad \forall t \tag{2b}$$

$$0 \le V^{DAg}_{ti} \le V^{bidG}_{ti} : \underline{\mu}^{Vg}_{ti}, \bar{\mu}^{Vg}_{ti}, \quad \forall t, \forall i \tag{2c}$$

$$0 \le V^{DAd}_{ti} \le V^{bidD}_{ti} : \underline{\mu}^{Vd}_{ti}, \bar{\mu}^{Vd}_{ti}, \quad \forall t,\ \forall i \tag{2d}$$

$$0 \le P^{G}_{tjb} \le \bar{P}^{G}_{tjb} + \Delta P^{G}_{tjb} : \underline{\mu}^{G}_{tjb}, \bar{\mu}^{G}_{tjb}, \quad \forall t,\ \forall j,\ \forall b \tag{2e}$$

$$0 \leq P^{D}_{tdk} \leq \bar{P}^{D}_{tdk} + \Delta P^{D}_{tdk} : \underline{\mu}^{D}_{tdk}, \bar{\mu}^{D}_{tdk}, \quad \forall t, \forall d, \forall k \tag{2f}$$

$$-\bar{C}_l \leq \sum_n H_{nl} \left(\sum_{(i\in\psi_n)} (V^{DAg}_{ti} - V^{DAd}_{ti}) + \sum_{(j\in\psi_n)} \sum_b P^{G}_{tjb} - \sum_{(d\in\psi_n)} \sum_k P^{D}_{tdk} \right) \leq \bar{C}_l$$
$$: \underline{\vartheta}_{tl}, \overline{\vartheta}_{tl} \quad \forall t, \forall l \tag{2g}$$

$$\lambda^{DA}_{tn} = \lambda^{DA}_{tf} - \sum_l H_{nl} \left(\overline{\vartheta}_{tl} - \underline{\vartheta}_{tl}\right), \quad \forall t, \forall n \tag{2h}$$

As it is seen in *Model (1)*, the objective function of the virtual bidder is maximized regarding to its main variables $\Omega = \{\alpha^{bidG}_{ti}, V^{bidG}_{ti}, \alpha^{bidD}_{ti}, V^{bidD}_{ti}, Ug_{ti}, Ud_{ti}\}$ and minimized with respect to the uncertain parameters $\Gamma = \{\Delta\lambda^{RT}_{tn}, \Delta P^{G}_{tjb}, \Delta P^{D}_{tdk}, \Delta\lambda^{G}_{tjb}, \Delta\lambda^{D}_{tdk}\}$. Constraints (1b) and (1c) set the maximum bounds for the virtual bids (generation/demand). Constraint (1d) guarantees that virtual generation and demand cannot be submitted to the DA market simultaneously. Uncertain parameters are limited in (1f) – (1j) by means of corresponding confidence intervals. The robust parameters ζ^{RT}_n, σ^{G}_{jb}, σ^{D}_{dk}, τ^{G}_{jb}, and τ^{D}_{dk} are determined by the virtual bidder and used as known parameters to measure the length of uncertain range around the predicted values. Note that the correlation between uncertain variables can be reflected in the corresponding bounds in this model. Furthermore, the model is flexible to take into account size-varying bounds of uncertain variables for different time periods.

It is worth mentioning that this paper focuses on the most influential uncertainties (such as predicted RT price, rivals' offers/bids) for the bidding strategy problem. Other events with low probabilities, such as unplanned outages of generating units and transmission branches, are ignored in this work.

Model (2) represents the lower-level subproblem which is linear since the ISO takes α^{bidG}_{ti} and α^{bidD}_{ti} and V^{bidG}_{ti} and V^{bidD}_{ti} as parameters. Therefore, it can be substituted by its KKT conditions. Combining these equivalenced constraints in the upper-level subproblem results in a Mathematical Problem with Equilibrium Constraint (MPEC), whose formulation is as follows.

III. EQUIVALENT MILP FORMULATION

A. MPEC MODEL (Model 3)

$$\underset{\Omega}{\text{Min}}\,\underset{\Gamma}{\text{Max}} \sum_t \sum_{(i\in\psi_n)} \left(\lambda^{DA}_{tn} - \left(\lambda^{RT}_{tn} + \Delta\lambda^{RT}_{tn}\right)\right) (V^{DAg}_{ti} - V^{DAd}_{ti}) \tag{3a}$$

Subject to:

$$\text{Constraints (1b)-(1j)} \tag{3b}$$

$$\alpha^{bidG}_{ti} - \lambda^{DA}_{tn} + \bar{\mu}^{Vg}_{ti} - \underline{\mu}^{Vg}_{ti} = 0, \quad \forall t, \forall i \in \psi_n \tag{3c}$$

$$-\alpha^{bidD}_{ti} + \lambda^{DA}_{tn} + \bar{\mu}^{Vd}_{ti} - \underline{\mu}^{Vd}_{ti} = 0, \quad \forall t, \forall i \in \psi_n \tag{3d}$$

$$\lambda^{G}_{tjb} + \Delta\lambda^{G}_{tjb} - \lambda^{DA}_{tn} + \bar{\mu}^{G}_{tjb} - \underline{\mu}^{G}_{tjb} = 0, \quad \forall t, \forall j \in \psi_n, \forall b \tag{3e}$$

$$-\lambda^{D}_{tdk} - \Delta\lambda^{D}_{tdk} + \lambda^{DA}_{tn} + \bar{\mu}^{D}_{tdk} - \underline{\mu}^{D}_{tdk} = 0, \quad \forall t, \forall d \in \psi_n, \forall k \tag{3f}$$

$$\text{Constraints (2b) and (2h)} \tag{3g}$$

$$0 \leq V^{DAg}_{ti} \perp \underline{\mu}^{Vg}_{ti} \geq 0, \quad \forall t, \forall i \tag{3h}$$

$$0 \leq V^{DAd}_{ti} \perp \underline{\mu}^{Vd}_{ti} \geq 0, \quad \forall t, \forall i \tag{3i}$$

$$0 \leq P^{G}_{tjb} \perp \underline{\mu}^{G}_{tjb} \geq 0, \quad \forall t, \forall j, \forall b \tag{3j}$$

$$0 \leq P^{D}_{tdk} \perp \underline{\mu}^{D}_{tdk} \geq 0, \quad \forall t, \forall d, \forall k \tag{3k}$$

$$0 \leq V^{bidG}_{ti} - V^{DAg}_{ti} \perp \bar{\mu}^{Vg}_{ti} \geq 0, \quad \forall t, \forall i \tag{3l}$$

$$0 \leq V^{bidD}_{ti} - V^{DAd}_{ti} \perp \bar{\mu}^{Vd}_{ti} \geq 0, \quad \forall t, \forall i \tag{3m}$$

$$0 \leq \bar{P}^{G}_{tjb} + \Delta P^{G}_{tjb} - P^{G}_{tjb} \perp \bar{\mu}^{G}_{tjb} \geq 0, \quad \forall t, \forall j, \forall b \tag{3n}$$

$$0 \leq \bar{P}^{D}_{tdk} + \Delta P^{D}_{tdk} - P^{D}_{tdk} \perp \bar{\mu}^{D}_{tdk} \geq 0, \quad \forall t, \forall d, \forall k \tag{3o}$$

$$0 \leq \bar{C}_l + \sum_n H_{nl} \left(\sum_{(i\in\psi_n)} (V^{DAg}_{ti} - V^{DAd}_{ti}) + \sum_{(j\in\psi_n)} \sum_b P^{G}_{tjb} - \sum_{(d\in\psi_n)} \sum_k P^{D}_{tdk} \right) \perp \underline{\vartheta}_{tl} \geq 0, \quad \forall t, \forall l \tag{3p}$$

$$0 \leq \bar{C}_l - \sum_n H_{nl} \left(\sum_{(i\in\psi_n)} (V^{DAg}_{ti} - V^{DAd}_{ti}) + \sum_{(j\in\psi_n)} \sum_b P^{G}_{tjb} - \sum_{(d\in\psi_n)} \sum_k P^{D}_{tdk} \right) \perp \overline{\vartheta}_{tl} \geq 0, \quad \forall t, \forall l \tag{3q}$$

Complementarity constraints related to inequality constraints are stated by (3h)–(3q) which are nonlinear equations, which can be linearized using the Fortuny-Amat transformation (Big M method) described in [29], [30]. Thus, each of the equations of $0 \leq V_{ti} \perp d_{ti}(x) \geq 0$ can be rewritten as follows.

$$0 \leq V_{ti} \leq M_{ti}\omega_{ti}, 0 \leq d_{ti}(x) \leq (1 - \omega_{ti}) M_{ti}$$

where M_{ti} is a large number and ω_{ti} is a binary variable. Therefore, the equivalent model will be as *Model (4)*.

B. EQUIVALENT MILP FORMULATION (Model (4))

$$\text{Objective Function (3a)} \tag{4a}$$

Subject to:

$$\text{Constraints (3b)–(3g)} \tag{4b}$$

$$\text{Linearized form of (3h)–(3q) based on Big M method} \tag{4c}$$

Now, the only nonlinear equation in *Model (4)* is the objective function, which is expressed explicitly with regard to uncertainties (Γ). To linearize the objective function, at the first step, it needs to be described implicitly based on Γ, which can be done using the SDT [15]. Due to the linearity of the inner problem, SDT can provide an objective function that has a zero duality-gap with the primal objective function value at the optimal point [31]. Doing some mathematical simplification, the objective function (4a) can be implicitly expressed with respect to the uncertain variables Γ as follows (equation (4a)):

$$\begin{aligned}\underset{\Omega}{\text{Min}}\,\underset{\Gamma}{\text{Max}} \sum_t \Bigg[&\sum_d \sum_k \left(\lambda_{tdk}^D + \Delta\lambda_{tdk}^D\right) P_{tdk}^D \\ &- \sum_i \left(\lambda_{tn}^{RT} + \Delta\lambda_{tn}^{RT}\right))(V_{ti}^{DAg} - V_{ti}^{DAd}) \\ &- \sum_j \sum_b \left(\lambda_{tjb}^G + \Delta\lambda_{tjb}^G\right) P_{tjb}^G \\ &- \sum_j \sum_b \bar{\mu}_{tjb}^G \left(\bar{P}_{tjb}^G + \Delta P_{tjb}^G\right) \\ &- \sum_d \sum_k \bar{\mu}_{tdk}^D \left(\bar{P}_{tdk}^D + \Delta P_{tdk}^D\right) \\ &- \sum_l \bar{C}_l \left(\overline{\vartheta}_{tl} + \underline{\vartheta}_{tl}\right) \Bigg]\end{aligned}$$

Therefore, *Model (4)* represents the single level nonlinear max-min problem. In order to remove the nonlinearities in the objective function, duality theorem is used here. Since the internal optimization problem (which is with regard to uncertain set) is linear, the dual form of that can be replaced. This procedure is fully illustrated in [32]. Employing this approach to *Model (4)* leads us to the following linear maximization form (*Model (5)*).

C. FINAL MODEL (Model (5))

$$\begin{aligned}&\underset{\Omega,\Phi}{\text{Max}}\, Z \\ &\Phi = \{\bar{\rho}_{tn}^{RT}, \underline{\rho}_{tn}^{RT}, \bar{\eta}_{tjb}^G, \underline{\eta}_{tjb}^G, \bar{\eta}_{tdk}^D, \underline{\eta}_{tdk}^D, \\ &\bar{\theta}_{tjb}^G, \underline{\theta}_{tjb}^G, \bar{\theta}_{tdk}^D, \underline{\theta}_{tdk}^D, \bar{\chi}_{tjb}^G, \underline{\chi}_{tjb}^G, \bar{\chi}_{tdk}^D, \underline{\chi}_{tdk}^D, \\ &\text{and all dual variables of the LL subproblem}\}\end{aligned} \tag{5a}$$

Subject to:

$$\begin{aligned}\sum_t \Bigg[&\sum_d \sum_k \left\{ \tau_{dk}^D \lambda_{tdk}^D \left(\bar{\theta}_{tdk}^D - \underline{\theta}_{tdk}^D\right) \right. \\ &+ \sigma_{dk}^D \bar{P}_{tdk}^D \left(\bar{\eta}_{tdk}^D - \underline{\eta}_{tdk}^D\right) \\ &\left. + \lambda_{tdk}^D P_{tdk}^D - \bar{\mu}_{tdk}^D \bar{P}_{tdk}^D \right\} \\ &+ \sum_i \left\{ \zeta_n^{RT} \lambda_{tn}^{RT} \left(\bar{\rho}_{tn}^{RT} - \underline{\rho}_{tn}^{RT}\right) \right. \\ &\left. - \lambda_{tn}^{RT} (V_{ti}^{DAg} - V_{ti}^{DAd}) \right\} \\ &+ \sum_j \sum_b \left\{ \tau_{jb}^G \lambda_{tjb}^G \left(\bar{\theta}_{tjb}^G - \underline{\theta}_{tjb}^G\right) \right. \\ &+ \sigma_{jb}^G \bar{P}_{tjb}^G \left(\bar{\eta}_{tjb}^G - \underline{\eta}_{tjb}^G\right) \\ &\left. - \lambda_{tjb}^G P_{tjb}^G - \bar{\mu}_{tjb}^G \bar{P}_{tjb}^G \right\} \\ &- \sum_l \bar{C}_l \left(\overline{\vartheta}_{tl} + \underline{\vartheta}_{tl}\right) \Bigg] \geq Z\end{aligned} \tag{5b}$$

$$\text{Constraints (4b) and (4c)} \tag{5c}$$

$$\bar{\rho}_{tn}^{RT} + \underline{\rho}_{tn}^{RT} = V_{ti}^{DAd} - V_{ti}^{DAg}, \quad \forall t, \forall i \in \psi_n \tag{5d}$$

$$sigma_{jb}^G \bar{P}_{tjb}^G \left(\bar{\chi}_{tjb}^G - \underline{\chi}_{tjb}^G\right) \geq P_{tjb}^G - \bar{P}_{tjb}^G, \quad \forall t, \forall j, \forall b \tag{5e}$$

$$\sigma_{dk}^D \bar{P}_{tdk}^D \left(\bar{\chi}_{tdk}^D - \underline{\chi}_{tdk}^D\right) \geq P_{tdk}^D - \bar{P}_{tdk}^D, \quad \forall t, \forall d, \forall k \tag{5f}$$

$$\sigma_{jb}^G \bar{P}_{tjb}^G \left(\bar{\chi}_{tjb}^G - \underline{\chi}_{tjb}^G\right) \leq \left(1 - \bar{\omega}_{tjb}^G\right) M^P + P_{tjb}^G - \bar{P}_{tjb}^G, \quad \forall t, \forall j, \forall b \tag{5g}$$

$$\sigma_{dk}^D \bar{P}_{tdk}^D \left(\bar{\chi}_{tdk}^D - \underline{\chi}_{tdk}^D\right) \leq \left(1 - \bar{\omega}_{tdk}^D\right) M^P + P_{tdk}^D - \bar{P}_{tdk}^D, \quad \forall t, \forall d, \forall k \tag{5h}$$

$$\tau_{jb}^G \lambda_{tjb}^G \left(\bar{\pi}_{tjb}^G - \underline{\pi}_{tjb}^G\right) = \lambda_{tn}^{DA} - \lambda_{tjb}^G - \bar{\mu}_{tjb}^G + \underline{\mu}_{tjb}^G, \quad \forall t, \forall j, \forall b \tag{5i}$$

$$\tau_{dk}^D \lambda_{tdk}^D \left(\bar{\pi}_{tdk}^D - \underline{\pi}_{tdk}^D\right) = -\lambda_{tn}^{DA} + \lambda_{tdk}^D - \bar{\mu}_{tdk}^D + \underline{\mu}_{tdk}^D, \quad \forall t, \forall d, \forall k \tag{5j}$$

$$\bar{\eta}_{tjb}^G + \underline{\eta}_{tjb}^G = -\bar{\mu}_{tjb}^G, \quad \forall t, \forall j, \forall b \tag{5k}$$

$$\bar{\eta}_{tdk}^D + \underline{\eta}_{tdk}^D = -\bar{\mu}_{tdk}^D, \quad \forall t, \forall d, \forall k \tag{5l}$$

$$\bar{\theta}_{tjb}^G + \underline{\theta}_{tjb}^G = -P_{tjb}^G, \quad \forall t, \forall j, \forall b \tag{5m}$$

$$\bar{\theta}_{tdk}^D + \underline{\theta}_{tdk}^D = P_{tdk}^D, \quad \forall t, \forall d, \forall k \tag{5n}$$

$$\bar{\chi}_{tjb}^G + \underline{\chi}_{tjb}^G = -1, \quad \forall t, \forall j, \forall b \tag{5o}$$

$$\bar{\chi}_{tdk}^D + \underline{\chi}_{tdk}^D = -1, \quad \forall t, \forall d, \forall k \tag{5p}$$

$$\bar{\pi}_{tjb}^G + \underline{\pi}_{tjb}^G = -1, \quad \forall t, \forall j, \forall b \tag{5q}$$

$$\bar{\pi}_{tdk}^D + \underline{\pi}_{tdk}^D = -1, \quad \forall t, \forall d, \forall k \tag{5r}$$

$$\left\{\bar{\rho}_{tn}^{RT}, \bar{\chi}_{tjb}^G, \bar{\chi}_{tdk}^D, \bar{\pi}_{tjb}^G, \bar{\pi}_{tdk}^D, \bar{\eta}_{tjb}^G, \bar{\eta}_{tdk}^D, \bar{\theta}_{tjb}^G, \bar{\theta}_{tdk}^D\right\} \leq 0$$

$$\left\{\underline{\rho}_{tn}^{RT}, \underline{\chi}_{tjb}^G, \underline{\chi}_{tdk}^D, \underline{\pi}_{tjb}^G, \underline{\pi}_{tdk}^D, \underline{\eta}_{tjb}^G, \underline{\eta}_{tdk}^D, \underline{\theta}_{tjb}^G, \underline{\theta}_{tdk}^D \geq 0\right\}$$

In Model (5), constraints (5b), (5d), and (5k) - (5n) are the dual forms of the objective function (4a) with respect to its corresponding constraints (1f) – (1j). Lagrangian coefficients of these constraints are $\bar{\rho}_{tn}^{RT}$, $\underline{\rho}_{tn}^{RT}$, $\bar{\eta}_{tjb}^G$, $\underline{\eta}_{tjb}^G$, $\bar{\eta}_{tdk}^D$, $\underline{\eta}_{tdk}^D$, $\bar{\theta}_{tjb}^G$, $\underline{\theta}_{tjb}^G$, $\bar{\theta}_{tdk}^D$, and $\underline{\theta}_{tdk}^D$. Constraints (5e) – (5j) are the dualized constraints of the primal constraints (2e), (2f), (3n), (3o), (3e), and (3f). Dualized equations of constraints (1g) – (1j) are

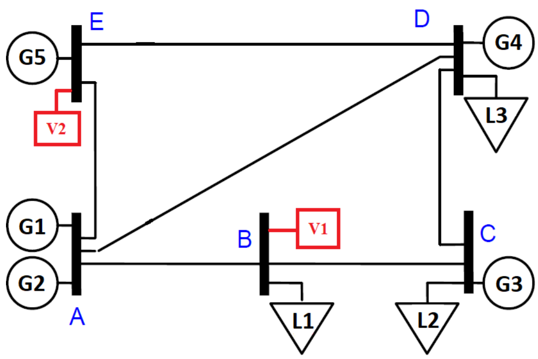

FIGURE 3. Five-bus test system.

TABLE 1. Forecasted RT price for different buses.

Bus # / Hour	A	B	C	D	E
1	12	50	30	45	10

TABLE 2. Forecasted generators offer quantities and prices.

	$\overline{P}_{tjb}^{G}$ (MW)	λ_{tjb}^{G} ($/MWH)	Location (Bus #)
G_1	40	14	A
G_2	170	15	A
G_3	520	30	C
G_4	200	40	D
G_5	600	20	E

stated as (5o) – (5r), respectively. Variables $\bar{\chi}_{tjb}^{G}$, $\underline{\chi}_{tjb}^{G}$, $\bar{\chi}_{tdk}^{D}$, $\underline{\chi}_{tdk}^{D}$, $\bar{\pi}_{tjb}^{G}$, $\underline{\pi}_{tjb}^{G}$, $\bar{\pi}_{tdk}^{D}$, and $\underline{\pi}_{tdk}^{D}$ are the Lagrangian coefficients of (1f) – (1j) to evaluate the dual of constraints (2e), (2f), (3e), (3f), (3n), and (3o). With this method, which is well described in [32], the robust two-level optimization problem is converted to a single level MILP problem which can be solved by available commercial solvers.

IV. ILLUSTRATIVE EXAMPLE

The considered test system is illustrated in Fig. 3. This system includes 5 generators, 3 loads, and 6 transmission lines. It is assumed that the virtual bidder attends to submit its bids from two locations (bus B and bus E) to the DA market.

For the sake of simplicity, it is assumed that the problem is solved for one period, and the corresponding forecasted RT prices ($/MWH) for different buses are shown in Table 1. Forecasted generators'/loads' offers/bids are summarized in Table 2 and Table 3, respectively.

Line capacities are assumed to be 400MW for the line A-B, 240MW for the line D-E, and 100MW for the rest of the lines. All robustness parameters are 0.1, and V_{ti}^{budget} is 200MW for virtual bids maximum offers/bids.

Solving the Deterministic model (Note that the Deterministic model can be obtained by setting all robustness parameters to zero), the virtual bidder's strategy would be 28.14MW generation at the price of $57.57/MWH at bus B, and 200MW generation at the price of $20/MWH at bus E. As it is shown in Table 4, if we test the Deterministic model results in the worst-case scenario, none of these virtual bids are cleared since, in this case, these bid prices are higher than the LMP of the system at the corresponding nodes.

TABLE 3. Forecasted loads bid quantities and prices.

	$\overline{P}_{tdk}^{D}$ (MW)	λ_{tdk}^{D} ($/MWH)	Location (Bus #)
L_1	300	60	B
L_2	300	60	C
L_3	400	75	D

TABLE 4. Deterministic and robust models results in the worst-case scenario.

		α_{ti}^{bidG} ($/MWH)	α_{ti}^{bidD} ($/MWH)	V_{ti}^{bidG} (MW)	V_{ti}^{bidD} (MW)	V_{ti}^{DAg} (MW)	V_{ti}^{DAd} (MW)
Deterministic	V_1	57.57	0	28.14	0	0	0
	V_2	20	0	200	0	0	0
Robust	V_1	0	66	0	19	0	19
	V_2	18	0	200	0	200	0

On the contrary, the bidding strategy of the virtual bidder is completely different when he/she applies the proposed model (*Model (5)*). As this model considers the occurrence of the worst-case scenario, its solution will be optimal in this scenario. The worst-case scenario happens when the L_1 and G_5 are the marginal MPs at buses B and E, respectively. As the robustness parameters are 0.1, the LMPs will be $66/MWH at bus B and $18/MWH at bus E, in the worst-case scenario. Therefore, virtual bidder bids as a demand at bus B at the price of $66/MWH and as a generator at bus E at the price of $18/MWH to be cleared in the DA market in this situation. As a result, the total profit of the virtual bidder is $404 using the proposed robust model at the worst-case scenario, while the profit would be zero when bids obtained from the Deterministic model are used.

V. CASE STUDY

A. DATA AND CASES SETUPS

The proposed approach is implemented on the IEEE 24-bus Reliability Test System [33] (Fig. 4). This system includes 24 buses, 32 generators, 17 demands, and 38 transmission lines. A virtual bidder is assumed to bid from 5 different locations (buses #6, #11, #14, #16, #22). Suppose the maximum bid quantity that virtual bidder can bid in the DA market is

TABLE 5. Forecasted offer quantities and prices of other generating units.

Gen #	$\overline{P}^{G}_{tjb}$ (MW)	λ^{G}_{tjb} ($/MWH)	Location (Bus #)	Gen #	$\overline{P}^{G}_{tjb}$ (MW)	λ^{G}_{tjb} ($/MWH)	Location (Bus #)
G_1	20	13.7	1	G_{17}	12	26.11	15
G_2	20	13.7	1	G_{18}	12	26.11	15
G_3	76	13.32	1	G_{19}	12	26.11	15
G_4	76	13.32	1	G_{20}	155	10.53	15
G_5	20	13.7	2	G_{21}	155	10.53	16
G_6	20	13.7	2	G_{22}	400	5.47	18
G_7	76	13.32	2	G_{23}	400	5.47	21
G_8	76	13.32	2	G_{24}	50	0	22
G_9	100	20.76	7	G_{25}	50	0	22
G_{10}	100	20.76	7	G_{26}	50	0	22
G_{11}	100	20.76	7	G_{27}	50	0	22
G_{12}	197	10.89	13	G_{28}	50	0	22
G_{13}	197	10.89	13	G_{29}	50	0	22
G_{14}	197	10.89	13	G_{30}	155	10.53	23
G_{15}	12	26.11	15	G_{31}	155	10.53	23
G_{16}	12	26.11	15	G_{32}	350	20.72	23

TABLE 6. Different cases design for uncertainties (%).

Uncertainty source	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8	Case 9
ζ^{RT}_{n}	0	20	0	0	0	0	10	20	30
σ^{G}_{jb}	0	0	20	0	0	0	10	20	30
σ^{D}_{dk}	0	0	0	20	0	0	10	20	30
τ^{G}_{jb}	0	0	0	0	20	0	10	20	30
τ^{D}_{dk}	0	0	0	0	0	20	10	20	30

60MW, which is determined according to the proxy amount owned by the virtual bidder [7]. Forecasted Real-Time LMP at different locations and periods are presented in Fig. 5. Offer quantities and prices of other generation units are represented in Table 5, which are assumed to be the same for all periods. Forecasted loads' bid quantities are depicted in Fig. 6, and their corresponding predicted bid prices is shown in Fig. 7. Note that this predicted bid price profile is considered the same for all loads.

We designed 9 different cases to present the effectiveness of the proposed model. The first case is the Deterministic case in which all robustness parameters are zero. In Cases 2 – 6, just one robustness parameter is assumed to be non-zero, and in other Cases (Cases 7 – 9), all robustness parameters are non-zero. Cases 7 – 9 are designed to present the benefit of the proposed robust model in the highly uncertain situation. Table 6 summarizes the designed cases.

B. RESULTS AND DISCUSSION

We solved the designed cases, explained in the previous section, with the proposed *Model (5)*. Moreover, for

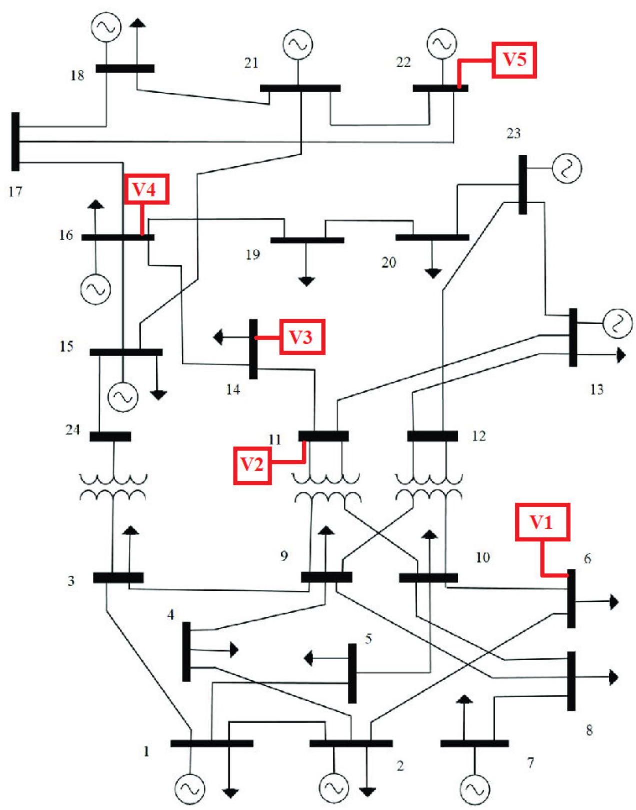

FIGURE 4. IEEE 24-bus test system.

comparison purpose, the Deterministic model results were tested at the worst-case situations. As it is shown in Fig. 8, the total profit of virtual bidder is always higher than the profit this MP can obtain from the Deterministic model. This is because the Deterministic model results are not applicable in the worst-case scenario and most of the time, they are not

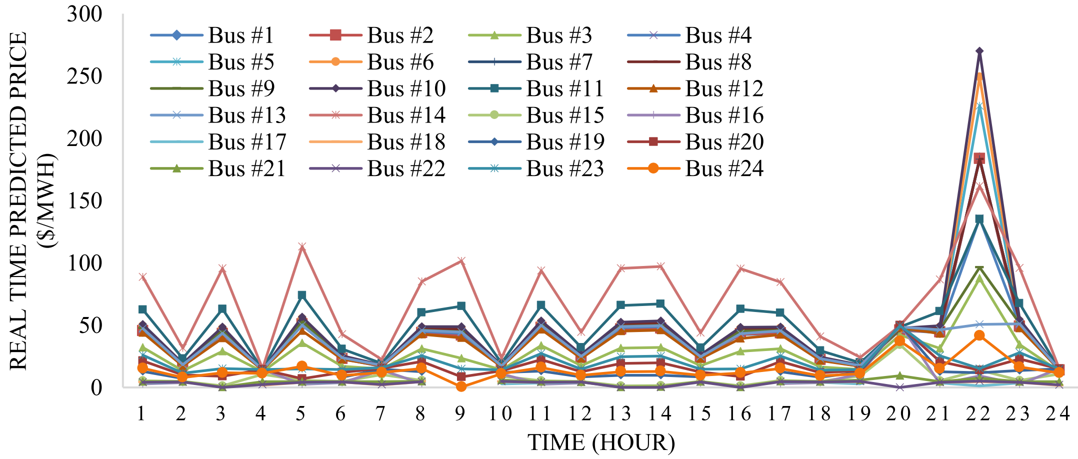

FIGURE 5. Forecasted RT market LMP at different buses and periods.

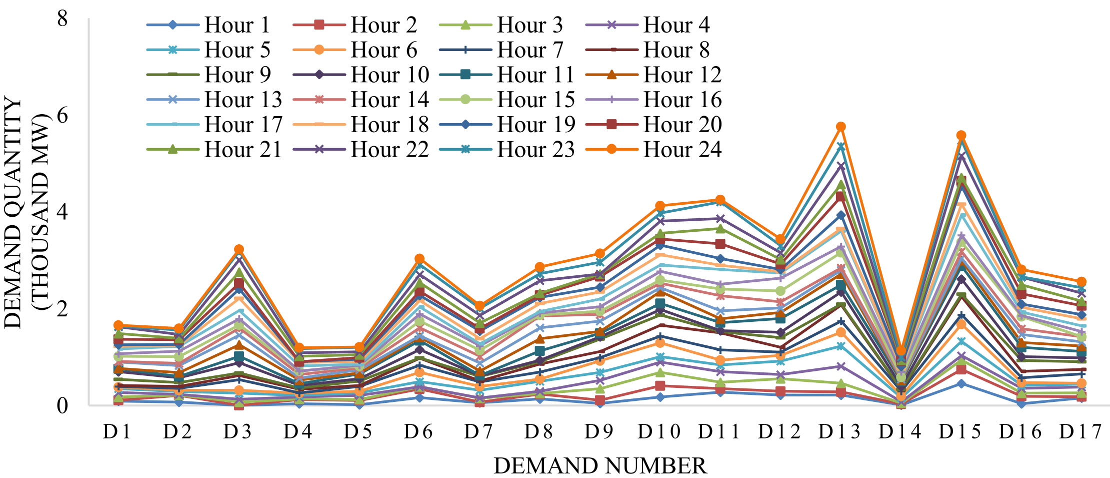

FIGURE 6. Forecasted loads quantities at different periods.

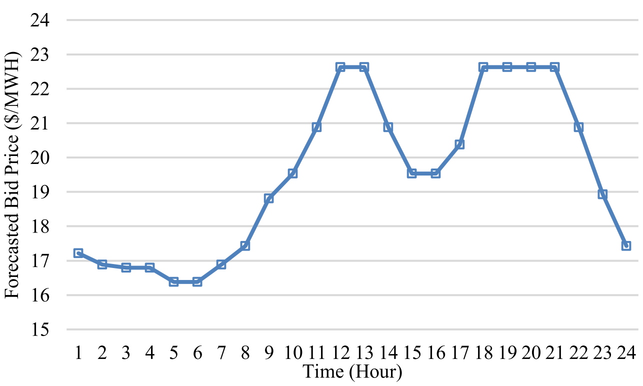

FIGURE 7. Forecasted bid prices for all loads at different periods.

cleared in the DA market, which leads to lower profit. Therefore, a risk-averse virtual bidder would prefer to apply the Robust-based solution in situations with uncertain sources.

All tests were performed on a computer with a 3.2 GHz Intel Core i7 CPU and 32GB of RAM. The models were implemented in AIMMS 4.75.1.0 [34] and solved using CPLEX 12.10 [35]. The number of variables, constraints, and CPU clock times regarding the deterministic model and robust model are summarized in Table 7.

TABLE 7. Number of variables, constraints and CPU clock times of the deterministic and robust models.

	Deterministic Model	Robust Model
# Variables	17041	29617
# Constraints	15025	25705
CPU Time	3.1 sec	17.4 sec

To present the influence of the virtual bids on the DA market prices, DA LMPs at two selected buses where the virtual bidder places its bids (buses 6 and 22), are shown in Fig. 9. Note that these prices are captured in the worst-case scenario. As seen from Fig. 9, the predicted RT LMP at bus 6 is higher than the DA LMP before placing the virtual bids. Using the deterministic model, virtual bids cause a reverse divergence between RT and DA LMPs at multiple hours, which results in a negative profit for the virtual bidder. However, there is a

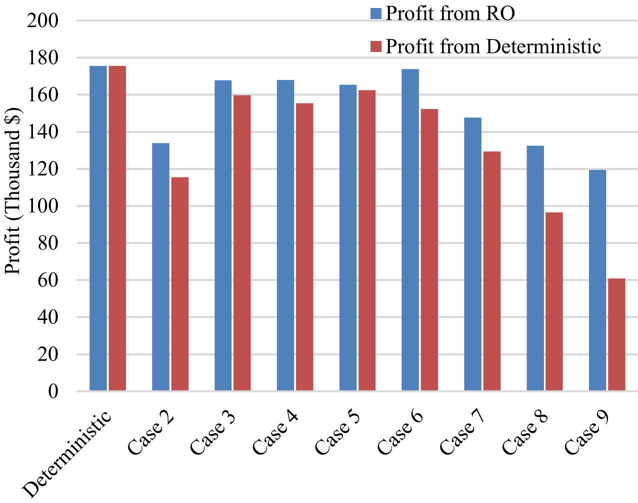

FIGURE 8. Total profit of the virtual bidder using deterministic and robust optimization at the worst-case scenario of different test cases.

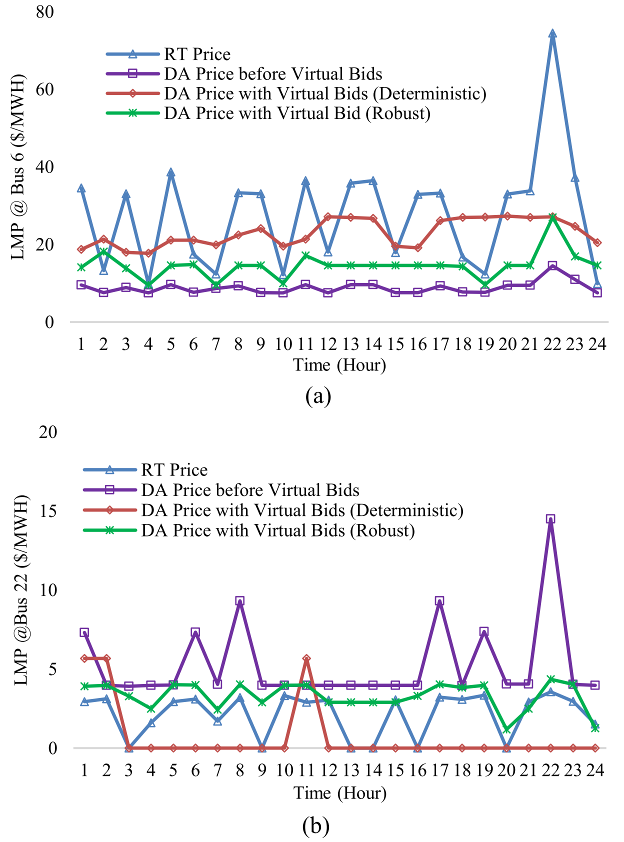

FIGURE 9. RT price, DA price before virtual bids, DA price with virtual bids using deterministic model, and DA price with virtual bids using the robust model at bus 6 (a) and bus 22 (b).

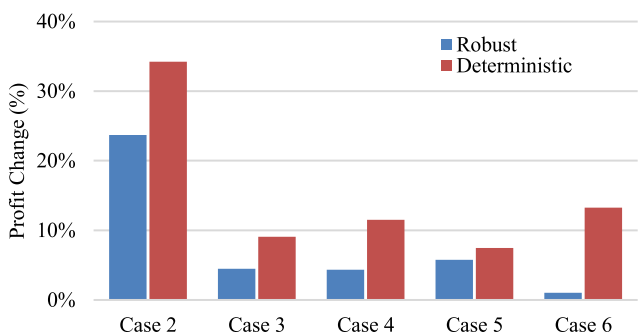

FIGURE 10. Profit change in cases 2–6.

reasonable convergence between RT and DA LMPs when the robust optimization results are applied by the virtual bidder. The same situation applies to LMPs at bus 22, except that the predicted RT price is smaller than the DA LMP before virtual bids.

A sensitivity analysis has been done here to find the most critical uncertain parameter which can highly affect the total profit. Therefore, the *Profit Change* is calculated using equation (6) for designed Cases 2 – 6. In each of the cases, only one of the uncertainty parameters is considered. In equation (6), R_d is the profit of Deterministic result in the forecasted scenario, and R_t is the profit of the Deterministic/Robust models' results testing in the worst-case scenario.

$$Profit\ Change = \frac{|R_d - R_t|}{R_d} \times 100 \tag{6}$$

As it is seen in Fig. 8, the profit obtained from the Deterministic result in the forecasted scenario (Case 1) is $175,635, while the profit of the virtual bidder is $133,974 when applying the Robust-based results in the worst-case scenario (Case 2). Thus, the profit change is 23.72% for this designed case. Fig. 10 compares the profit changes calculated for Cases 2 – 6 for both deterministic-based and Robust-based results tested at the worst-case scenarios. It is obvious that the higher the profit change is, the greater the impact of the corresponding parameter on the total profit. Therefore, as shown in Fig. 10, RT LMP has the greatest influence on the total profit of the virtual bidder.

In order to observe the performance of the proposed model, this model has been tested with different levels of uncertainty (Cases 7 – 9). As shown in Fig. 11, the difference between the profits attained from the proposed model and deterministic model results will rise as the level of uncertainty increases. Note that, in these tests, the worst-case scenario was utilized to evaluate the results of the robust and deterministic models. Therefore, the outcome of deterministic model results may change when the uncertainty level changes.

The *improvement in profit*, which is calculated by equation (7), represents the advantages of applying the proposed model specifically for the risk-averse virtual bidders who consider the higher confidence interval for the uncertain parameters. Fig. 11 shows that the improvement in profit reaches 50% when the virtual bidder chooses 0.3 for the robustness parameters in his/her decision-making process. It clearly demonstrates the benefits of the proposed model. In equation (7), R_r is the profit of the robust-based model,

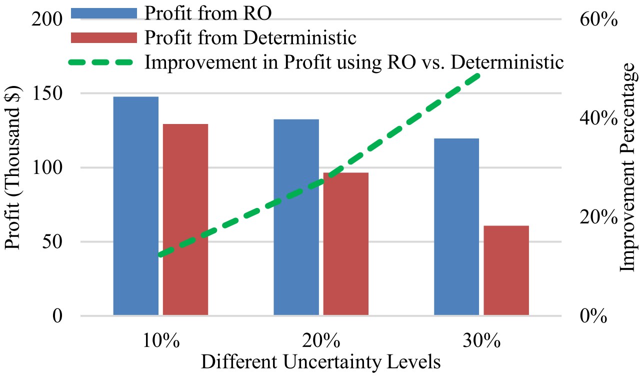

FIGURE 11. Profit comparison between the deterministic model and robust model results tested at the worst-case scenario with different level of uncertainty.

and R'_d is the profit of the Deterministic model results testing at the worst-case scenario.

$$Improvement\ in\ profit = \frac{|R_r - R'_d|}{R_r} \times 100 \tag{7}$$

VI. CONCLUSION

In this paper, a max-min two-level optimization model is presented to optimize the bidding strategy for a risk-averse virtual bidder taking part in the DA market. Duality theorem, KKT optimality conditions, SDT, and the big-M method are employed to translate the two-level problem into a MILP problem. As the lower-level subproblem of the model represents the quasi DA market, a virtual bidder can mimic the market clearing process and can appropriately make bidding decisions to its best interest. Therefore, through the bid price, a virtual bidder can effectively compromise between the amount of cleared virtual bids and the affected price difference between the DA and RT markets considering the uncertainties of other MPs' strategies and RT market LMPs. Numerical results and sensitivity analysis show that RT LMP is the most critical uncertain parameter that the virtual bidder needs to consider in his/her decision-making procedure. Moreover, as compared to using the deterministic model, a risk-averse virtual bidder can always make more profit at the worst-case scenario employing the proposed model, and the improvement in profits increases dramatically as the uncertainty level increases.

REFERENCES

[1] W. W. Hogan, "Virtual bidding and electricity market design," *Electr. J.*, vol. 29, no. 5, pp. 33–47, Jun. 2016.
[2] J. Birge, A. Hortaçsu, I. Mercadal, and M. Pavlin. (May 2013). *Virtual Bidders and Financial Transmission Rights in the MISO Electricity Market*. [Online]. Available: https://www.editorialexpress.com
[3] J. Kazempour and F. Benjamin Hobbs, "Value of flexible resources, virtual bidding, and self-scheduling in two-settlement electricity markets with wind generation—Part I: Principles and competitive model," *IEEE Trans. Power Syst.*, vol. 33, no. 1, pp. 749–759, Jan. 2018.
[4] M. Celebi, A. Hajos, and P. Q. Hanser, "Virtual bidding: The good, the bad and the ugly," *Electr. J.*, vol. 23, no. 5, pp. 16–25, Jun. 2010.
[5] S. D. Ledgerwood and J. P. Pfeifenberger, "Using virtual bids to manipulate the value of financial transmission rights," *Electr. J.*, vol. 26, no. 9, pp. 9–25, Nov. 2013.
[6] C. Lo Prete, N. Guo, and U. V. Shanbhag, "Virtual bidding and financial transmission rights: An equilibrium model for cross-product manipulation in electricity markets," *IEEE Trans. Power Syst.*, vol. 34, no. 2, pp. 953–967, Mar. 2019.
[7] W. Wang and N. Yu, "A machine learning framework for algorithmic trading with virtual bids in electricity markets," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Aug. 2019, pp. 1–5.
[8] D. Xiao, W. Qiao, and L. Qu, "Risk-averse offer strategy of a photovoltaic solar power plant with virtual bidding in electricity markets," in *Proc. IEEE Power Energy Soc. Innov. Smart Grid Technol. Conf. (ISGT)*, Feb. 2019, pp. 1–5.
[9] H. Mehdipourpicha and R. Bo, "Risk-constrained bi-level optimization for virtual bidder bidding strategy in day-ahead electricity markets," in *Proc. IEEE Power Energy Soc. Gen. Meeting (PESGM)*, Aug. 2020, pp. 1–5.
[10] A. J. Conejo, F. J. Nogales, and J. M. Arroyo, "Price-taker bidding strategy under price uncertainty," *IEEE Trans. Power Syst.*, vol. 17, no. 4, pp. 1081–1088, Nov. 2002.
[11] J. M. Morales, A. J. Conejo, and J. PÉrez-Ruiz, "Short-term trading for a wind power producer," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 554–564, Feb. 2010.
[12] E. G. Kardakos, C. K. Simoglou, and A. G. Bakirtzis, "Optimal bidding strategy in transmission-constrained electricity markets," *Electr. Power Syst. Res.*, vol. 109, pp. 141–149, Apr. 2014.
[13] A. G. Bakirtzis, N. P. Ziogos, A. C. Tellidou, and G. A. Bakirtzis, "Electricity producer offering strategies in day-ahead energy market with step-wise offers," *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 1804–1818, Nov. 2007.
[14] V. P. Gountis and A. G. Bakirtzis, "Bidding strategies for electricity producers in a competitive electricity marketplace," *IEEE Trans. Power Syst.*, vol. 19, no. 1, pp. 356–365, Feb. 2004.
[15] C. Ruiz and A. J. Conejo, "Pool strategy of a producer with endogenous formation of locational marginal prices," *IEEE Trans. Power Syst.*, vol. 24, no. 4, pp. 1855–1866, Nov. 2009.
[16] S. Shafiee, P. Zamani-Dehkordi, H. Zareipour, and A. M. Knight, "Economic assessment of a price-maker energy storage facility in the Alberta electricity market," *Energy*, vol. 111, pp. 537–547, Sep. 2016.
[17] L. Baringo and A. J. Conejo, "Offering strategy of wind-power producer: A multi-stage risk-constrained approach," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1420–1429, Mar. 2016.
[18] C. Boonchuay and W. Ongsakul, "Optimal risky bidding strategy for a generating company by self-organising hierarchical particle swarm optimisation," *Energy Convers. Manage.*, vol. 52, no. 2, pp. 1047–1053, Feb. 2011.
[19] M. Rahimiyan and L. Baringo, "Strategic bidding for a virtual power plant in the day-ahead and real-time markets: A price-taker robust optimization approach," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2676–2687, Jul. 2016.
[20] X. Han, E. G. Kardakos, and G. Hug, "A distributionally robust bidding strategy for a wind power plant," *Electr. Power Syst. Res.*, vol. 177, Dec. 2019, Art. no. 105986.
[21] X. Han and G. Hug, "A distributionally robust bidding strategy for a wind-storage aggregator," *Electr. Power Syst. Res.*, vol. 189, Dec. 2020, Art. no. 106745.
[22] A. Baringo and L. Baringo, "A stochastic adaptive robust optimization approach for the offering strategy of a virtual power plant," *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3492–3504, Sep. 2017.
[23] S. Darvishi, G. Sheisi, and J. Aghaei, "Bidding strategy of hybrid power plant in day-ahead market as price maker through robust optimization," *Int. Trans. Elect. Energy Syst.*, vol. 30, no. 7, 2020, Art. no. e12426.
[24] B. Zeng, H. Dong, R. Sioshansi, F. Xu, and M. Zeng, "Bilevel robust optimization of electric vehicle charging stations with distributed energy resources," *IEEE Trans. Ind. Appl.*, vol. 56, no. 5, pp. 5836–5847, Sep. 2020.
[25] S. Shafiee, H. Zareipour, and A. M. Knight, "Developing bidding and offering curves of a price-maker energy storage facility based on robust optimization," *IEEE Trans. Smart Grid*, vol. 10, no. 1, pp. 650–660, Jan. 2019.
[26] H. Khajeh, A. Asghar Foroud, and H. Firoozi, "Robust bidding strategies and scheduling of a price-maker microgrid aggregator participating in a pool-based electricity market," *IET Gener., Transmiss. Distrib.*, vol. 13, no. 4, pp. 468–477, 2018.

[27] W. Wei, D. Wu, Z. Wang, S. Mei, and J. P. S. Catalao, "Impact of energy storage on economic dispatch of distribution systems: A multi-parametric linear programming approach and its implications," *IEEE Open Access J. Power Energy*, vol. 7, pp. 243–253, 2020.

[28] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton, NJ, USA: Princeton Univ. Press, 2009.

[29] J. Fortuny-Amat and B. McCarl, "A representation and economic interpretation of a two-level programming problem," *J. Oper. Res. Soc.*, vol. 32, no. 9, pp. 783–792, Sep. 1981.

[30] X. Wang *et al.*, "Tri-level scheduling model considering residential demand flexibility of aggregated HVACs and EVs under distribution LMP," *IEEE Trans. Smart Grid*, early access, May 28, 2021, doi: 10.1109/TSG.2021.3075386.

[31] A. J. Conejo and C. Ruiz, "Complementarity, not optimization, is the language of markets," *IEEE Open Access J. Power Energy*, vol. 7, pp. 344–353, 2020.

[32] J. M. Morales, A. J. Conejo, H. Madsen, P. Pinson, and M. Zugno, *Integrating Renewables in Electricity Markets: Operational Problems*, vol. 205. New York, NY, USA: Springer, 2013.

[33] C. Ordoudis, P. Pinson, J. M. Morales, and M. Zugno, "An updated version of the IEEE RTS 24-bus system for electricity market and power system operation studies," Tech. Univ. Denmark, Lyngby, Denmark, Tech. Rep., 2016. [Online]. Available: https://orbit.dtu.dk/en/publications/an-updated-version-of-the-ieee-rts-24-bus-system-for-electricity-

[34] *AIMMS*. Accessed: Sep. 2020. [Online]. Available: https://www.aimms.com

[35] (2008). *The ILOG CPLEX*. [Online]. Available: http://www.ilog.com/products /cplex/

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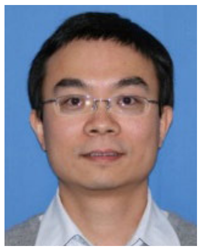

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